UNIT 12 FRACTIONS OF WHICH WHOLE ?

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12.1 INTRODUCTION

Although children meet fractions in everyday life they usually have very confused ideas about the real implication of the concept. For example, in conversation they often wrongly use the word 'half' when the word 'part' should have been used. We can only have half a cake if the cake has been divided into two equal parts. But, in practice, each piece is only **approximately** half of the cake. If we buy half a kilogram of tea, we have simply bought a packet of tea. In this case, the word, 'half' is only a label to many children. Another common misconception among children regarding half is that it can be obtained only when we divide a single whole, i.e., when the whole is a single object, and not when the whole is a set of objects. Children find it difficult to divide a whole in halves. Usually, they continue to divide, not stopping at two parts. They think that one cut produces one part and two cuts produce two parts. It is also seen that children when dividing an object say, cake for instance, take out two small pieces, leaving a large part undivided. This is because of the absence of any relation between the parts and the whole. **If the child has half, he cannot conclude that the other part is also half, as this also involves relating the part to the whole.** Why do children have confused ideas? In this unit, we have suggested some teaching methods which may help learners to overcome these confusions.

Objectives

After studying this unit, you should be able to

- use several child-centred methods for teaching the concept of a fraction;
- evaluate the efficacy of the method used;
- do remedial teaching whenever necessary.

12.2 IS HALF REALLY HALF ?

Many of us would have encountered situations like the one that happened at Renu's house. Her two children, Meena aged 10 and Ravi aged 8, were having their food. They both finished but one chapati was still left. So Renu asked both of them to share the chapati and finish it. Ravi took the chapati cut a corner of it and gave it to his sister. I asked Ravi, how much has he given to his sister and he promptly replied 'half'. Then I took a biscuit, cut a corner of it and gave it to him. I again asked him how much he has got and his reply was the same, 'half'. So, for Ravi every piece of a whole was 'half'.

Sonu, here is your half a glass of milk.

This is a very common problem occurring with the children of this age group. Any piece of a whole is half for them. But why does this misconception exist? One reason for this could be that right through their childhood they have seen their mother giving them half a chapati or half a glass of milk in the same manner. But out of curiousity I visited and observed a 3rd standard class, where the teacher was starting

to teach 'Fractions', using the NCERT book. The teacher drew various figures as in Fig. 1.

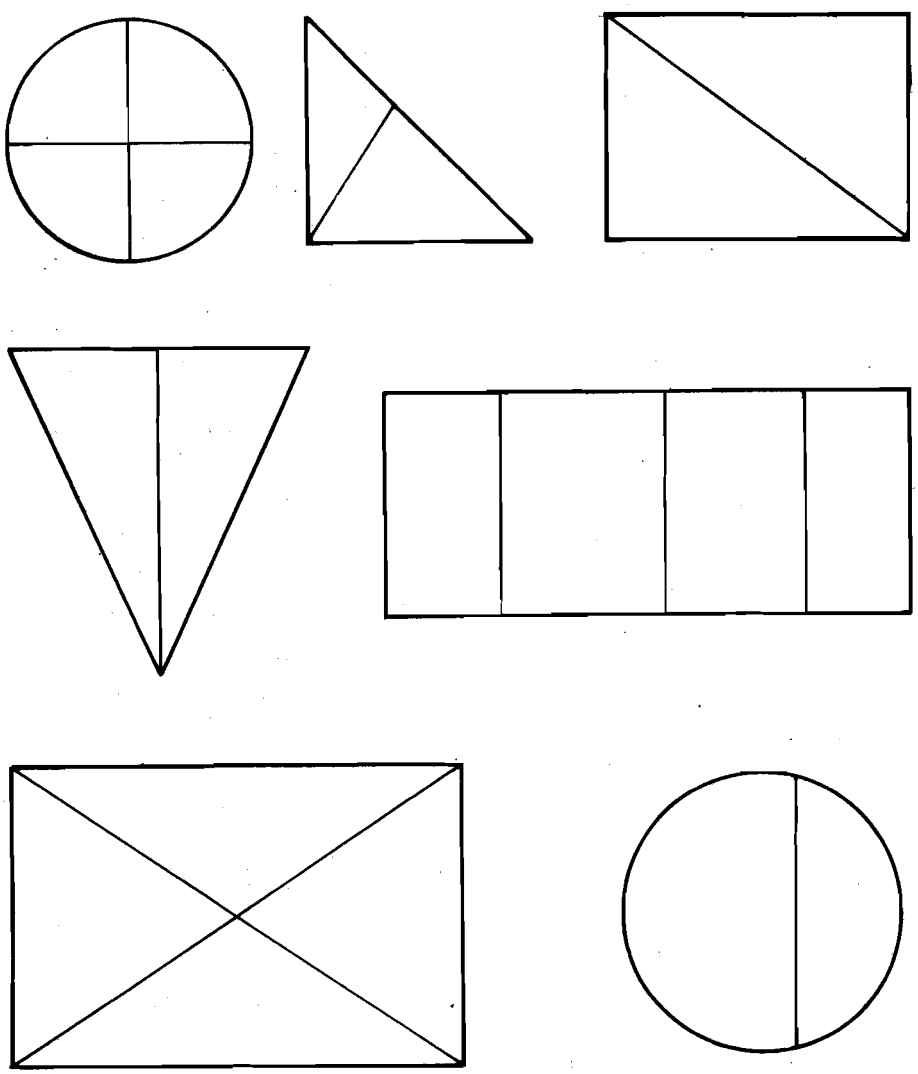

Fig.1

Then she asked the class, which of these figures were divided into two equal parts, and which were divided into four equal parts. The response of the class was quite good as most of the students came out with the correct answers.

Then she drew some more figures like those in Fig. 2.

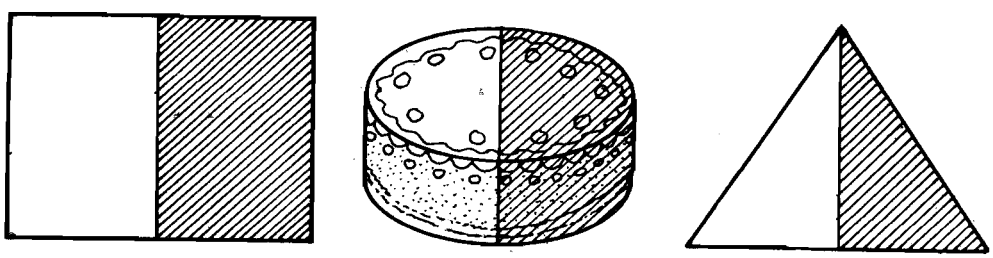

Fig.2

Each figure was divided into two equal parts. One shaded and the other unshaded. She told everyone that each of these shaded and unshaded portions were one-half of the whole and we call each of these parts as 'one-half'. Then she took one paper, folded it by bringing two opposite edges together and cut it into two equal pieces (see Fig.3).

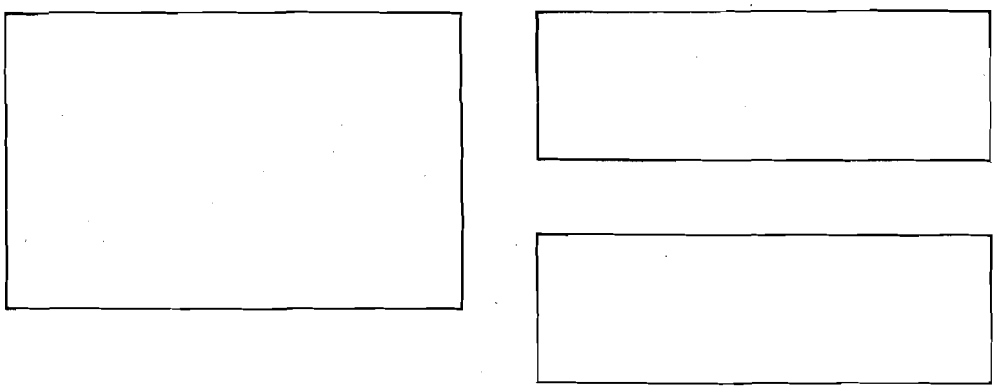

Fig.3

She called two students and gave a piece each to both of them. Then she asked the whole class how much each of them had? The entire class answered 'one-half'. Then she took both the pieces and put them one over the other to show that both the pieces were equal. In the same way she introduced the concept of one-fourth (or a quarter). Then she gave questions from the book to be done in the class and some questions for "Home work". The teacher looked quite satisfied. It appeared that she succeeded in doing whatever she intended doing. But I was a bit doubtful. I was not sure that the importance of having **two equal parts** for finding 'one-half' of any whole, really got through to the children. Also that these two equal parts can be obtained in a number of ways, i.e., it is not always necessary to have the dividing line as a straight line.

I thought I would follow up my observations with some evaluation to gauge how much communication actually took place between the teacher and the learners. One of the children, Anita, was a daughter of a good friend of mine. A couple of days later I went to Anita's place. After a bit of casual conversation, I showed Anita five figures as shown in Fig 4.

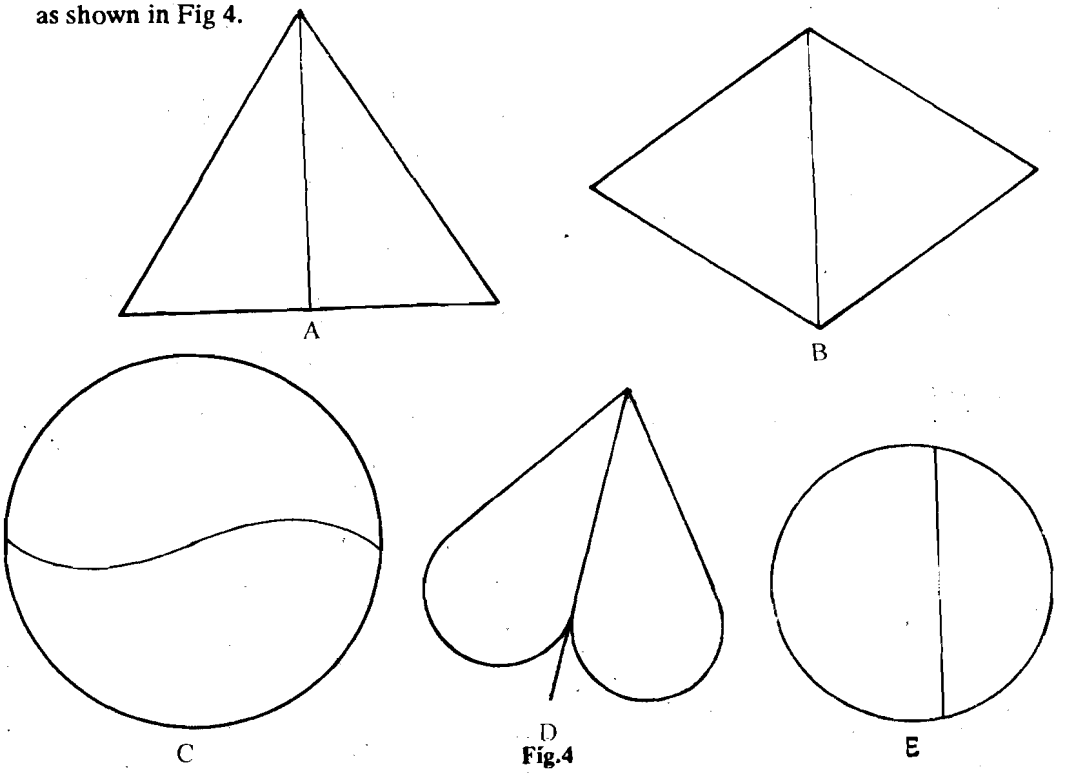

Fig.4

Example 1

I : Anita, tell me which of the following figures are divided into two equal parts (equal halves).

Anita : Fig. A B and E are divided into two equal parts.
Fig. C is not and um.... D is also divided into two equal parts.

I : Are you very sure that C is not divided into two equal parts and E is divided into equal parts?

Anita : Yes Auntie, I am very sure.

I : O.K., let's take a paper cutting in the shape of C. I will cut it along the curved line drawn. So now we have these two cuttings (see Fig.5).

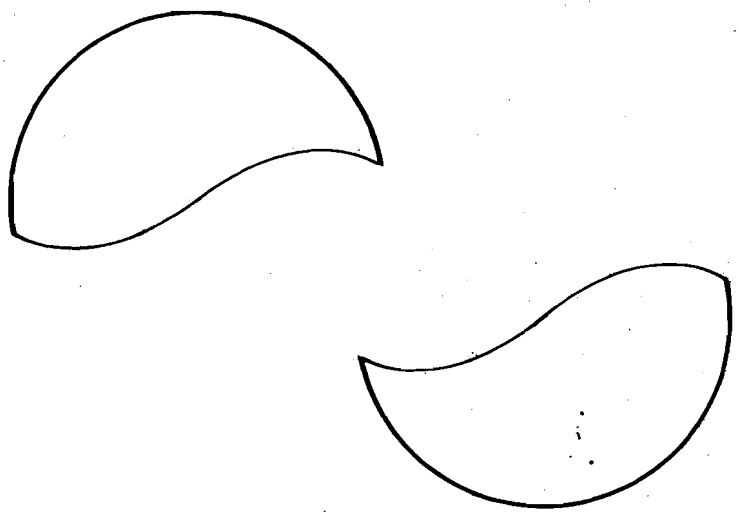

Fig.5

Now, what happens if you put one cutting over another?

Does the first cutting cover the second one completely?

Anita : Yes Auntie.

I : That means the two cuttings are equal.

Anita : Oh! yes, even these two parts are of the same size. Now I know C is also divided into two equal parts.

I felt happy, as Anita herself came out with the answer. Then I took up Fig. E with her. Although difference in the two parts in E was not too obvious but, I could convince her with the help of a paper cutting that the two parts are not equal. I took up two more figures like the one shown in Fig. 6 with her. But now Anita

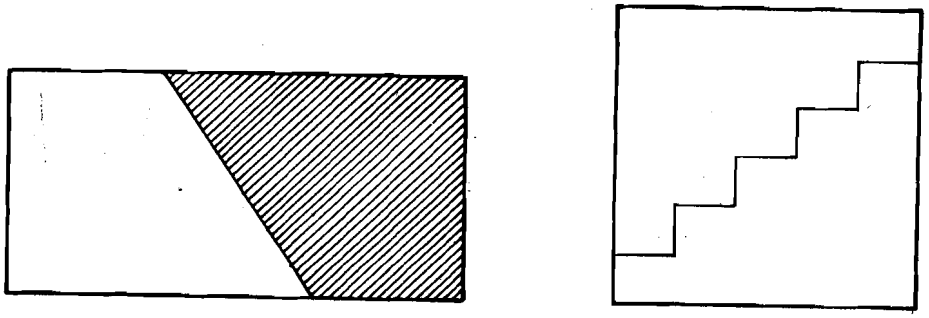

Fig.6

looked very confident and took no time in giving me the correct answers.

Why do you think Anita thought C had not been divided in half.

Anita's problem about E was not unique. It is usually seen that unless it becomes too obvious, as in Stage IV, Fig.7, a child would continue saying upto Stage III, that the

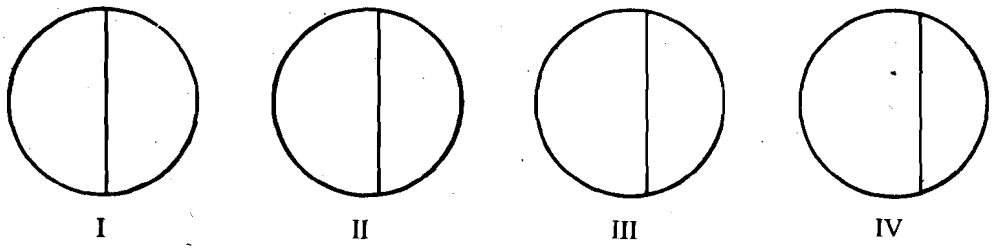

Fig.7

two parts are equal. This shows that while introducing 'one-half' of a whole in the class, stress should be given on getting two equal parts and it should be shown to the children when actually two parts are equal. In this connection here we suggest two simple activities which a teacher can take up in the class.

Two parts are equal if one put over the other covers it completely.

Example 2 : Take one big piece of paper and 3-4 smaller pieces of papers. Ask children to find which pieces are halves of the bigger paper.

Example 3 : Take a ribbon. Ask two boys in the class to share it between themselves so that each of them get half a ribbon. Once they have done it you can make them check whether the two pieces are equal.

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Also, a teacher needs to involve the learners in some more activities like dividing different shapes as circle, square or a triangle into two equal parts in 3 or 4 ways. Children should not only experience dividing a square, triangle, rectangle etc. into two equal parts. They should also realise that if there are ten cars in a race and five fail to finish then **half the number** of cars have failed to finish.

To start with, you could introduce situations where equal sharing is possible. For example, if two children share 10 sweets equally, each has one-half of the sweets, so one-half of 10 equals 5. Ask the children to put out a set of 6 or 8 or 12 bottle tops, chalk pieces or, other objects and ask them to remove half of them. How many objects have they? Discuss it with them.

E1) List two other activities to check that the importance of having two equal parts for finding half of any whole has really got through to children.

The concept of $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$, etc. can be introduced in the same manner.

Here are some suggestions for activities, that you may find useful.

Example 4 : Take a paper cutting of any of the shapes in Fig. 8.

Fold it by bringing two opposite edges together. Press it at the fold. Now unfold the paper. You will find that it has been divided by a line into two equal parts i.e., each part when laid one on top of the other, covers the other one completely. Each part is one half of the whole. Repeat the activity by folding the paper once again the same

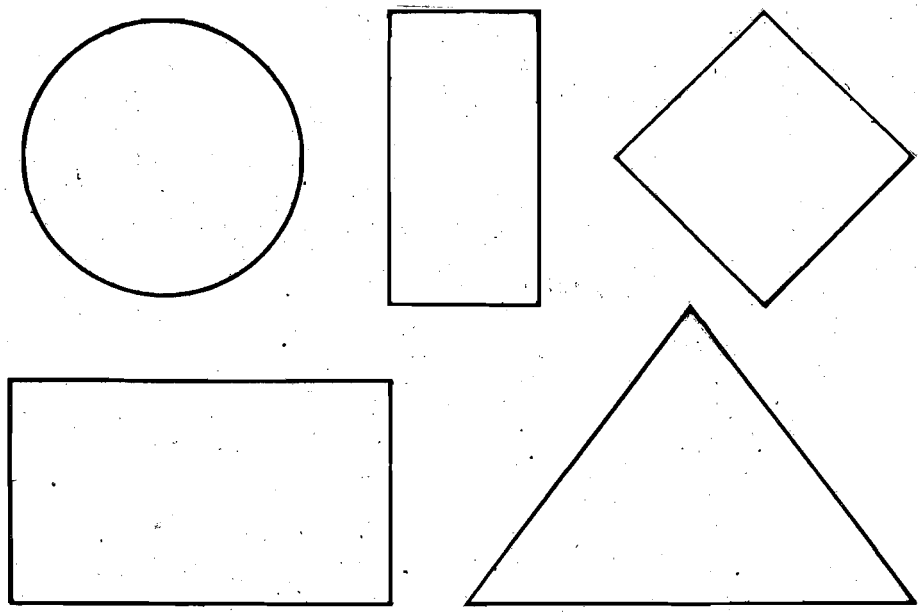
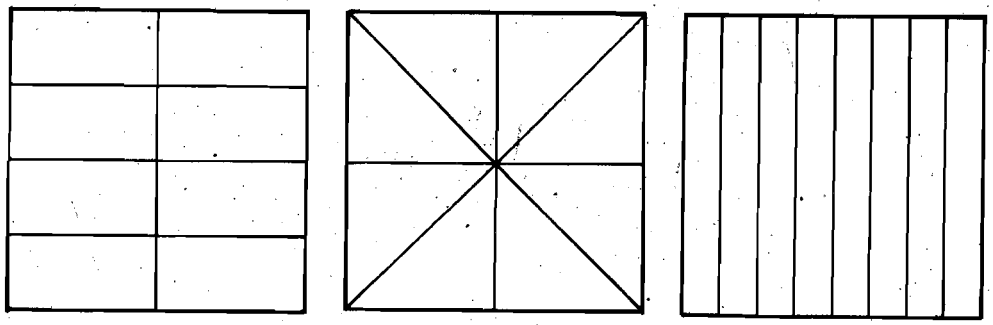


Fig.8

way and the paper is divided into four quarters. Continuing in this way you could familiarise them with $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, etc.

Note that the square and rectangle can be folded into eighths in different ways, e.g. see Fig.9.

Square



Rectangle

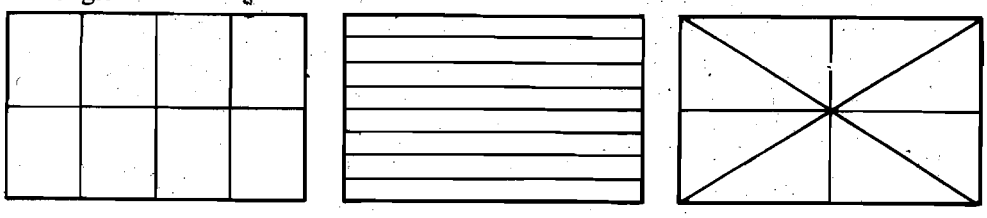


Fig.9

Children find this activity easy and interesting to do. But they may not find it easy to extend this activity to show 1/3. In this case you can help the students in folding the paper into 3 equal parts and then show $\frac{1}{3}, \frac{1}{6}, \ldots$ etc.

Let's take the case of Ankit who is an active student of mathematics. Ankit, came to ask me some of his doubts in mathematics. Ankit studies in the fourth standard and likes to do mathematics. While talking to him I discovered that they had just finished doing fractions in their class. So I thought of asking him to solve a problem. I asked him to write the fraction of a whole corresponding to the shaded portion in each of the three figures I drew (see Fig.10).

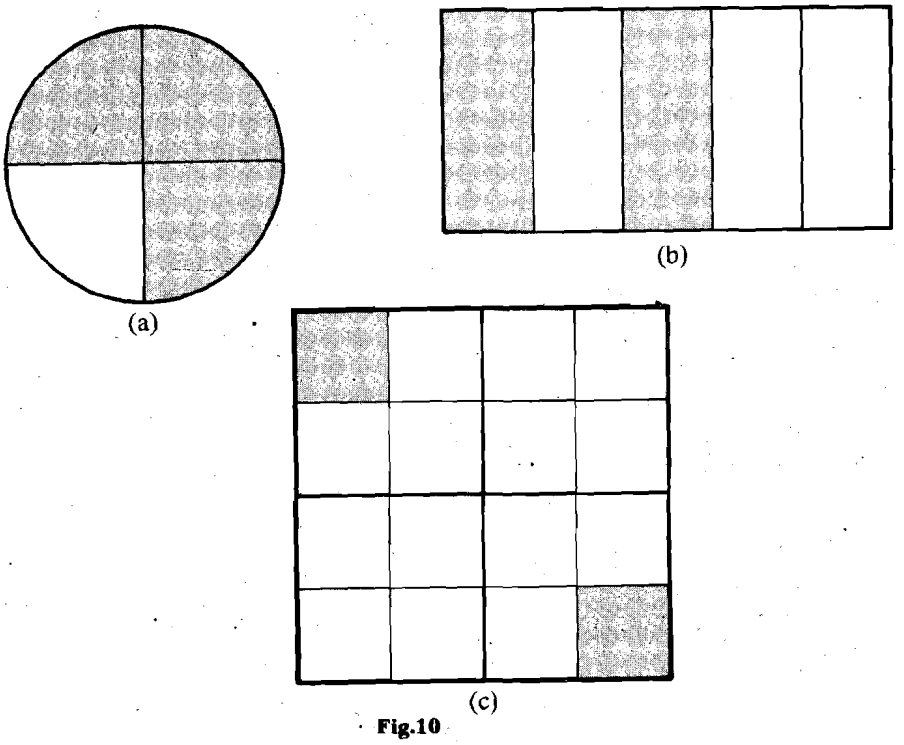


Fig.10

His answer to the problem was (a) $\frac{3}{4}$, (b) $\frac{2}{5}$, (c) $\frac{1}{4}, \frac{1}{4}$.

E2) a) What do you think Ankit's problem was?

b) Is this a very common problem with children?

c) How would you help him clarify his concepts?

E3) How would you help a child show $\frac{1}{3}, \frac{1}{6}$ by paper folding?

E4) Can you suggest an activity to help a child get the idea of $\frac{1}{5}$.

E5) How will you evaluate the success of your method of teaching fractions in your class?

What you may have realised from the above observation is that relating a part to a whole is another point of confusion that children face. We will address this problem in the following section.

12.3 WHOLE AND PART

The obstacle for understanding fractions is that it cannot be thought of as a separate, independent entity. **Fractions have meaning only in relation to the whole to which**

they apply. To recognize a fraction of something, we need a concept of the whole. You would agree with me when I say that three quarters (three-fourths) may be the expression of a single whole being divided into four equal parts, of which three have been chosen; or it may be the expression of the fact that there were four members/objects in a set out of which three members/objects have been taken; or, somewhat differently, it may imply that three whole have been divided into four equal parts. While these aspects lead to the same numerical result, you should bear in mind that they are different concepts and children need guidance in relating to them.

After talking to few teachers and parents it is felt that when children are given a single whole it is easy for them to understand the concept of 'part' and 'whole'. But when they are asked to find the part of a whole consisting of a collection of objects, say ten cars or five chairs, then they find it difficult to write the part of this whole. We could overcome such problems if we always refer to the whole to which any fraction applies. We should not talk about a quarter, but about a quarter of an apple, a quarter of a metre, or a quarter of twelve, and so on. Also, to start with we can introduce this concept restricting our discussion to a single whole.

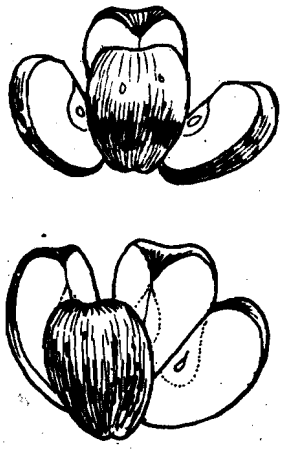


Fig. 11

In this regard some activities may be undertaken by the teacher in the class. Here is an activity, that was observed in a class, which I would like to share with you. Consider the following situation.

Example 5

Teacher : Here is an apple cut into four equal pieces/parts, When I put all these four parts together (showing it), I get the whole of the apple. (see Fig. 11)

Teacher : Mohan, come here and take one piece. What part of an apple have you got?

Mohan : One

Teacher : Out of how many parts?

Mohan : One out of four parts.

Teacher : Can you tell me what fraction of an apple have you got?

Mohan : Yes, madam, I have one-fourth of an apple.

Teacher : O.K., now take this second piece also. How much have you got now?

Mohan : I have got two pieces now.

Teacher : Out of how many pieces?

Mohan : I have two out of four.

Teacher : You have two out of four pieces. Now can you say this in terms of halves?

Mohan : Yes, Madam, I have got half of an apple now.

Teacher : Take one more piece and tell me the part of an apple you have got

Mohan : Now, I have three pieces out of four.

Teacher : That means you have three-fourths of an apple now.

Mohan : Yes, Madam.

Teacher : Mohan, now has three parts of an apple out of four parts, and these four parts, when put together make one whole apple. So this is how we obtain three-fourths of a whole. The whole in this case being an apple.

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The teacher then involved all the children in various activities with different objects like piece of paper, piece of bread, biscuit, chapati etc., whatever she could gather in her class. She showed $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$, to all the children with the help of these objects. In between she asked them a few ques ons. After she made them answer all the questions she looked quite satisfied.

As we have mentioned earlier children have problems in relating a part to a whole when the whole is a collection of objects. It is therefore necessary that we acquaint a learner with different possible whole. Once we feel that children are confident about a part-whole relationship with respect to a 'whole' consisting of a single object we can extend our discussion to other types of 'whole'.

Let's take up once again the problem of three-fourth of a whole and see how many other types of 'whole' we can acquaint a learner with.

1. A whole consisting of more than one but fewer than four objects

Three cakes to be shared equally by four children.

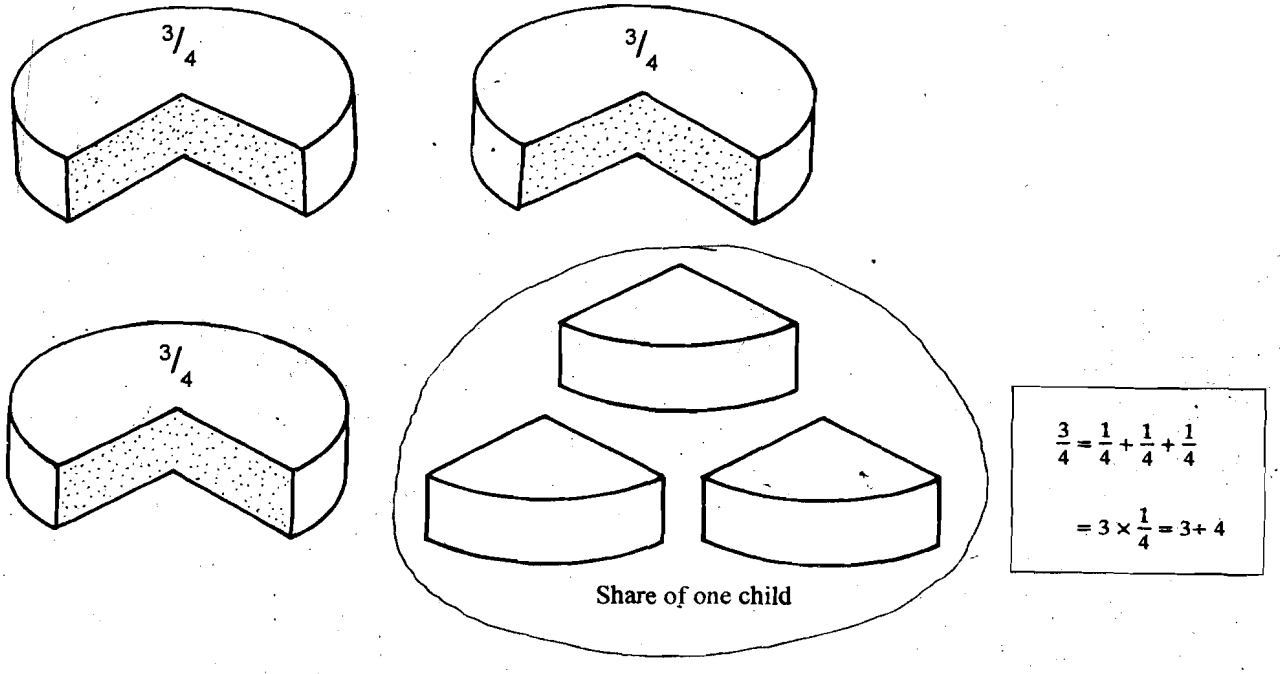


Fig:12

2. **A whole consisting of exactly four objects**

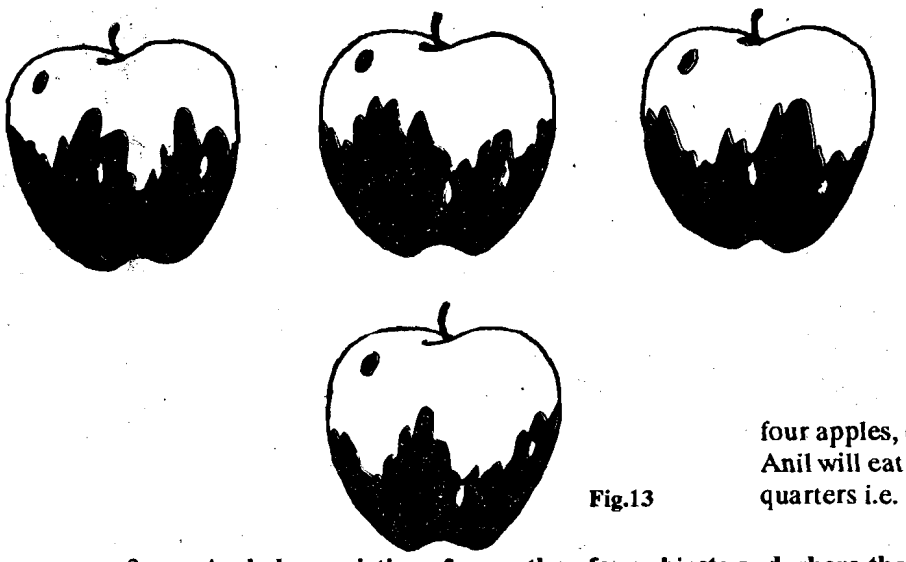
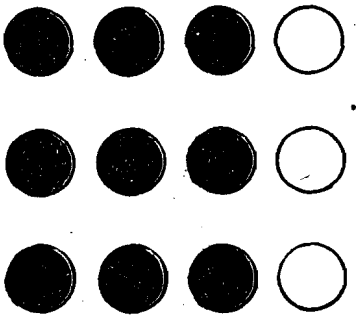


Fig.13

four apples, of which Anil will eat three quarters i.e. three

3. **A whole consisting of more than four objects and where the total is exactly divisible by four**



Twelve marbles, three quarter of which are black i.e., nine are black.

Fig.14

Teacher can take up activities in the class with each of the 'whole' mentioned above. Here we give one for you. Let's look at the following situation.

Example 6 : Teacher brought some biscuits and toffees in the class and started interacting with the children. She called one boy Mohan near her.

Teacher : Now here are four biscuits and I give three biscuits to Mohan, then how much has he got?

Students : He has got three biscuits.

Teacher : Out of how many?

Students : Out of four.

Teacher : Mohan, can you write on the board the fraction of biscuits you have got.

Mohan : Yes, madam. (He wrote $\frac{3}{4}$ on the board.)

Teacher : So, can we say that he has three-fourths of biscuits.

Students : Yes, Madam.

Teacher : So three-fourths in this case is three biscuits out of four biscuits. Can you tell me what is our whole in this case?

After a bit of silence in the class.

One boy : It's biscuits.

Teacher : How many biscuits?

Boy : Four biscuits.

Teacher : Good! So, our whole here is a set of objects namely, four biscuits.

Teacher again took five toffees and gave two toffees to Renu. Then she did similar exercise with the children. She asked them many questions and made them say that the 'Whole' in this case is five toffees taken together. Then she did few more exercises with them and gave them some problems to solve.

This way students can be exposed to different wholes.

E6) Similarly, suggest one activity each for other types of wholes mentioned above.

We now end this unit by giving a summary of what we have discussed in it.

12.4 SUMMARY

In this unit we have tried to bring the following points.

1) Children aged between 8 to 10 years usually have very confused ideas about 'half'. Every piece of a whole is half for them. They do not realise the importance of having two equal parts for finding 'one-half' of any whole. Also, for many of them the dividing line has necessarily got to be a straight line. Such misconceptions could be avoided to a great extent if children are given various activities in the class with the help of different concrete objects easily available around them. They can be involved in dividing different shapes such as circle, square or a triangle etc. into two equal parts in 3 or 4 ways. Once they divide a thing into halves they can be asked to check if both the parts are equal.

2) Relating a part to a whole is another major problem with the children of this age group. They do not realise that fractions have meaning only in relation to the whole to which they apply. To overcome such problems we should always refer to the whole to which any fraction applies. It is also necessary that we acquaint a learner with different possible wholes by taking up various activities in the class.

12.5 COMMENTS ON EXERCISES

E1) i) Give a piece of bread, or a chapati to a child and ask her to take half of it. After she does it let her check that both the halves are equal.

ii) Children can be given shapes of flat clay or paper and asked to divide them up between two, either by cutting, folding, or drawing.

E2) a) Ankit viewed at the four squares (with darker boundaries) within the bigger square as four wholes. He looked at them separately and answered. He could not visualise the four squares as part of the same whole i.e. the bigger square. It is a problem of relating part to a whole.

b) Yes, part-whole view of a fraction is a very common problem with children of this age group.

c) His concepts can be clarified by undertaking paper folding activities, or using various concrete objects like marbles, biscuits, toffees etc. for performing activities.

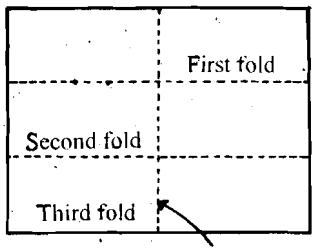


Fig.15

E3) You can take a rectangular piece of paper and fold it into three equal folds. Press it at the folds and then unfold the paper to show $\frac{1}{3}$. Repeating the same activity and folding the paper once again $\frac{1}{6}$ can be shown. (see Fig.15)

E4) Children usually find it difficult to divide a circle into five equal parts. So you can draw a circle or a rectangle, divide it into five equal parts as in Fig.16. Now shade one part out of these five parts and ask the child the fractional representation of it, or you may ask him to shade $\frac{1}{5}$ of it. You can also show it by paper folding. You can even take 10 sweets ask the child to distribute them equally among 5 friends. Now let the child say how much everybody got.

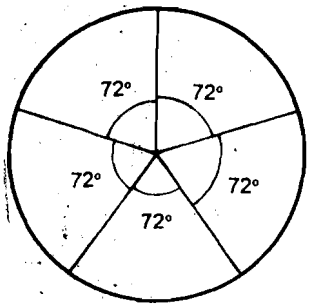


Fig.16

E5) You can first do paper sheet exercise with all the children in the class. Divide them into three groups. Ask each group to fold the paper into two equal parts, unfold and shade what corresponds to $\frac{1}{2}$. They should also be able to say that they have got $\frac{1}{2}$ of the whole. Then you may go to three parts which is much more difficult than folding into two parts. After this you may also give each group 12 marbles/toffees and ask each group to divide them into two, three or four equal parts.

E6) i) Take a dozen of bananas/oranges, three of which are rotten. Ask children to separate rotten bananas/oranges from the good ones. Now ask them the fractional representation of the remaining bananas/oranges.

ii) Take 3 chapatis and ask 4 boys to share them equally. They can do it in two ways

a) Three boys can take 1 chapati each and then each of them can give $\frac{1}{4}$ of a chapati to the fourth boy.

b) They can divide each of 3 chapatis into 4 equal parts and then share them equally.

After they have shared the chapatis equally they can be asked to write what fraction have they got.

UNIT 13 TYPES OF FRACTIONS

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13.1 INTRODUCTION

It is usually seen that children's perception of a fraction is very restricted. They find it virtually impossible to think of a fraction as a number and consequently are unable to relate it with any number. If you ask a child what 'half' means, more often than not she will say that it is a part of a cake, or a bun, or some such object. This kind of answer is typical of what she would say for one-third, one-fourth, or any other fraction. Children fail to recognise any link between fractions and numbers. This could be due to the restricted and imprecise use of language in describing fractions. They might have never heard their teacher saying that fractions are numbers. They see fractions only in terms of their concrete representation and hence are unable to name it or symbolise it.

In this unit we have suggested some activities that you can use to help children in understanding fractions as abstract quantities and how to symbolise them. We have also talked about equivalence and ordering of fractions.

Objectives

After studying this unit you should be able to

- use teaching methods that help children to recognise a link between fractions and numbers and use notations for naming a fraction ;
- use various child-centred methods to teach the concept of equivalent fractions ;
- use teaching methods that help children identify fractions greater than one, less than or equal to one, and list them in required order.

13.2 NAMING FRACTIONS

The other day I was talking to my niece Ankita and her friend Rama both students of Class IV. Just then I remembered my friend a primary teacher who once told me that children find it difficult to name fractions. I thought of testing it on these two girls. Look at this situation

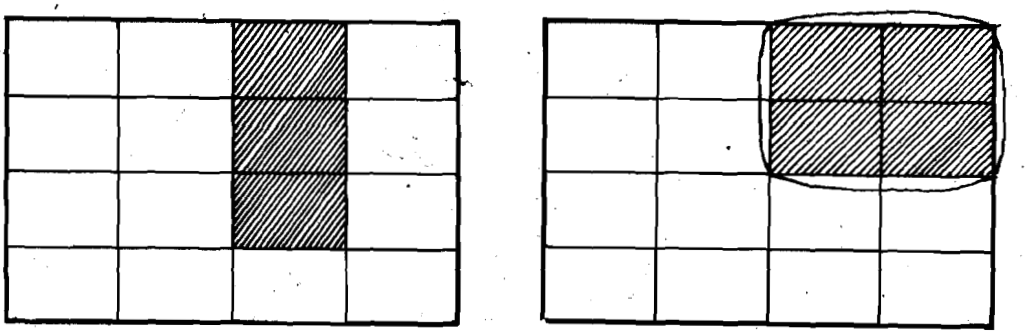
Example 1

I : What fraction is the shaded region in each case?

Ankita : 1(a) is three-sixteenth and 1(b) is four-sixteenth.

Rama : 1(a) is three-sixteenth and 1(b) is one-fourth.

It was really interesting to get these answers. I thought of assessing Rama's depth of understanding of naming fractions.



1(a) 1(b)

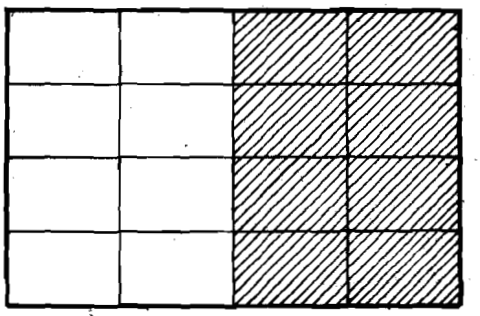
Fig.1

I : Rama why do you say that 1(b) is one-fourth?

Rama : (A bit hesitantly). Because there's ..... there's sixteen on a sheet, and there's four each.... each side of the .... . If you split it up then that's a fourth and that's a fourth and that's a fourth (pointing to sets of four squares at a time).

But as I watched her a doubt crept into my mind. Because her action as she said 'and there's four each.... each side of the ...' indicated that she was actually counting the four rectangles in the ringed region. Was she saying the ringed region was a fourth because there were four rectangles in it, or because there were four areas of the same size on the whole sheet? This doubt was confirmed when Ankita and Rama tried to explain why a certain area was one-eighth.

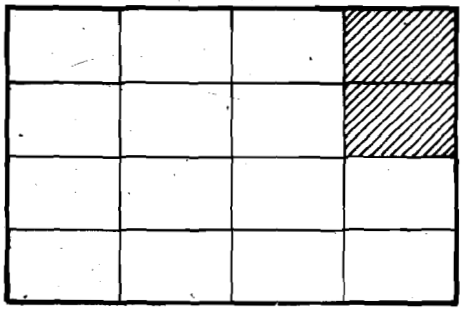
Rama :



It's one eighth because it's got eight squares.

Fig.2

Ankita :



No, this is one-eighth because the whole can be split into eight pieces like this.

Fig.3

So, naming a fraction and understanding the notation by which fractions are symbolised is another big problem with children.

E1) Which of the illustrations and explanations above is correct? What would you do to focus on the correct solution?

Any confusion in a child's mind about the procedure by which a fraction is named should be cleared before proceeding further with the topic. When children are thoroughly familiar with fraction names such as a 'half', 'third', 'quarter', and with the meaning of those names in relation to various wholes, for instance, a third of a cake, they are ready to be introduced to symbols for fractions. It is seen that we usually tell them that in order to write 'two-third' they can draw a line under '2' and write '3' beneath the line as $\frac{2}{3}$ without explaining why it is written this way. Children often get confused and write $\frac{3}{2}$ for two-thirds. We need to tell them that the numeral at the bottom of a fraction (the denominator) shows the total number of parts into which the whole is divided, and the numeral at the top (the numerator) is the number of parts that we are considering. For instance, the fraction $\frac{2}{3}$ tells us that the 'whole has been divided, into three equal parts, and we are looking at two of those parts. The denominator of the fraction $\frac{2}{3}$ gives the fraction its name 'third' and the numerator tells us the number of thirds to be considered.

We need to stress the writing of fractions as numbers. You could help children associate a number with the given fraction. For instance, a paper is divided into two equal parts and one part out of these two parts is represented by a number $\frac{1}{2}$. Children can visually see and link numbers with the given fraction.

They also need a lot of practice in translating fraction names to fraction symbols. My experience is that, to start with it is best to consider only fractions whose numerators are '1'. This helps to focus their attention on the significance of the denominator. Children might be asked to complete a table like this one:

Number of equal parts of the whole	2	3	4	5	6
Name of each part	1 half	1 third	1 quarter	–	–
Symbol for one part	$\frac{1}{2}$	$\frac{1}{3}$	–	–	–

Gradually, some more exercises/activities involving different numerators can be done by the children.

E2) Can you suggest some group activities for children in a class of 40, to see if the concept of naming and symbolising a fraction has got across to them?

Once children realise that fractions are numbers they may wonder about operations on them. But before you teach them how to add and subtract fractions, it is necessary that they have a clear understanding of other concepts like equivalence and ordering. These concepts play a central role in understanding addition and subtraction. Many teachers have found that children do not understand these concepts easily. In the next section we shall give some suggestions for communicating the idea of equivalent fractions to a child.

13.3 EQUIVALENCE OF FRACTIONS

Children do come across many instances where the same fraction can be named in

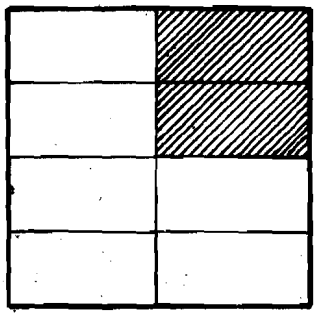


Fig. 4

more than one way. For example, many children say that $\frac{2}{8}$ of the square in Fig.4 is shaded and many others say that $\frac{1}{4}$ of the square is shaded.

Both these answers are correct. But children sometimes are not ready to accept the fact that different fractions may represent the same measure or equal amount of whatever object they are talking about. For example, with units attached, the fractions $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots$ represent equal measures. In this respect they are therefore equivalent. This concept requires a lot of practice and children take a long to grasp this idea.

Let me share my experience with you in this regard. Once I got a chance to interact with four children aged between 8 and 10 years in my neighbourhood. I took them for a treat to a nearby cafe. They all wanted big pastries. Consider this situation.

Example 2

I : If I buy 3 pastries how would you share them equally among yourselves?

One child : I will cut each of the three pastries into four equal parts and then give one part from each pastry to each of us. So, each of us will get an equal amount.

Another child : No, first I would give one pastry each to three of us and then ask each of these three to give one-fourth of their pastry to the fourth person. Then each of us will get an equal share.

I : O.K. Both of you are dividing 3 pastries into four equal shares like this (Fig.5).

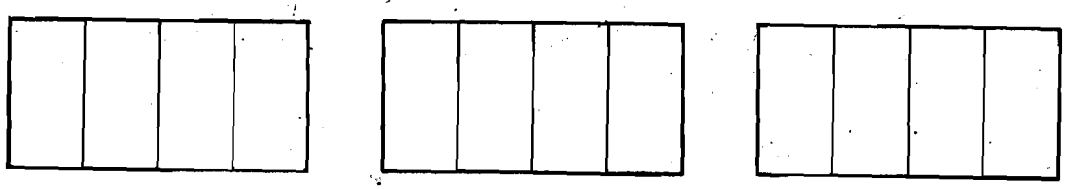


Fig.5(a)

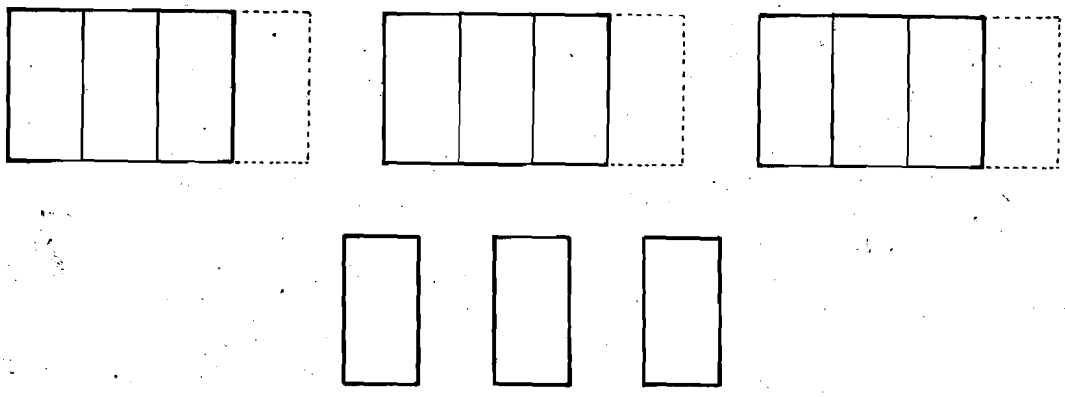


Fig.5 (b)

Why don't you all write down the fraction of pastry that each of you will get?

1st child : Each will get $\frac{1}{4}$

2nd child : Each will get $\frac{3}{4}$

3rd child : Each will get $\frac{3}{12}$

? What would you say about the responses of these children? Who was wrong and who was correct, and why?

In the above situation the first child looked at the three pastries as a single whole and so he got $\frac{1}{4}$ of it. For the second child the whole consisted of 3 discrete objects —the pastries and his share was $\frac{1}{4}$ of these three pastries. The third child again viewed at three pastries as a single whole divided into 12 equal parts out of which he got 3 parts.

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So from the above observations you would agree that it is necessary that children realise the need to clearly state what 'whole' they are referring to while naming a fraction. Once they are clear about it they will not have problems in representing the same measure by different fractional numbers.

To start with we can ask children without introducing any formal name, to do 'detective work' on a particular fraction to see how many other names they can find for it. But this detective work needs to be structured. You could start with the following activity.

Example 3 : Take a strip of paper fold it into eighths. Then invite children to find another way of naming one-half of the strip. Let them find that

$\frac{1}{2}$ of the strip is $\frac{2}{4}$ of the strip, and also $\frac{4}{8}$ of the strip.

In the same way they could divide a square into eighths and find that

$\frac{1}{2}$ of the square is $\frac{2}{4}$ of the square, and also $\frac{4}{8}$ of the square.

At this point you could explain the equivalence of one-half, two-fourths, and four-eighths, etc. and write $\frac{1}{2} = \frac{2}{4}$, which means that one-half is the same as two-fourths. You can ask the children to record the general findings they have made above by writing

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$$

Children can study the above list, and may notice a pattern in the denominators and numerators of the fractions in particular, that the denominator of each fraction is twice the numerator. You can ask children to suggest other fractions that follow the same pattern. You yourself can initiate and suggest $\frac{6}{12}$.

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Once, the children have got enough experience of finding the fractions equivalent to a given fraction and have observed the pattern followed in each case, let them try and discover the rule for writing equivalent fractions, namely, **multiply the numerator and denominator by the same number or divide the numerator and denominator by their common factor.**

E3) Suggest one activity each to help children find fractions equivalent to one-third, one-fourth and two-thirds.

Here is another activity which a teacher took up in her class to teach the concept of equivalent fraction using different concrete objects.

Example 4

Teacher : Here I have 32 marbles. I am giving 16 marbles each to Mohan and Sohan (two students in the class)

Now Mohan, you divide these 16 marbles into four equal parts.

Mohan : (Dividing like ○○ ○○ ○○ ○○ / ○○ ○○ ○○ ○○

and showing it to the teacher) Here are four equal parts.

Teacher : How many marbles are there in one part?

Mohan : 4 marbles.

Teacher : What fraction of marbles is this one part?

Mohan : It is $\frac{1}{4}$ of 16 marbles.

Teacher then calls another student Rani.

Teacher : Mohan, give two parts out of four to Rani.

Mohan : O.K. madam (giving ○○ ○○ / ○○ ○○ i.e. two parts to Rani).

Teacher : Mohan, tell me, what fraction of marbles have you given to Rani?

Mohan : I have given her $\frac{2}{4}$ of 16 marbles.

Teacher : Fine! Now Sohan, you divide your 16 marbles into 8 equal parts.

Sohan : (Dividing like ○○ ○○ ○○ ○○ ○○ ○○ ○○ ○○

and showing to teacher) Here I have divided them.

Then the teacher made him answer questions like how many marbles are there in one part? What fraction of marbles is this one part? Then she calls another girl Rita and tells Sohan to give her four parts out of 8. Sohan gave 4 parts to Rita. Fig. 4. ○○ ○○ ○○ ○○

Teacher : Sohan, tell me, what fraction of marbles have you given to Rita?

Sohan : I have given her $\frac{4}{8}$ of 16 marbles.

Teacher : Alright! So Rani has got $\frac{2}{4}$ of 16 marbles and Rita has got $\frac{4}{8}$ of 16 marbles. How many marbles each one of you have? (addressing to Rani and Rita).

Rani : 8 marbles madam.

Rita : I also have 8 marbles.

Teacher : Does it mean that $\frac{2}{4}$ of 16 marbles is the same as $\frac{4}{8}$ of 16 marbles?

Entire class roared: Yes Madam.

Teacher : Yes, you are right. The only thing is they are grouped differently.

In the same way she explains that if she has 2 chapatis split into 4 (see Fig.6)

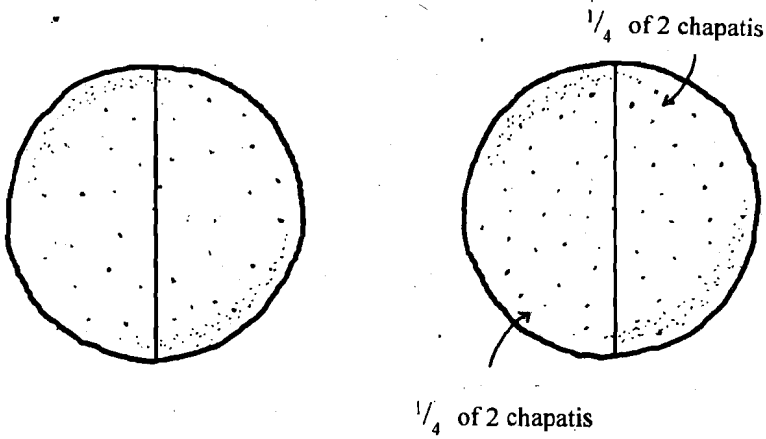

Fig.6

then $\frac{2}{4}$ of 2 chapatis is 1 whole chapati and if she splits 2 chapatis into 8 (see Fig.7)

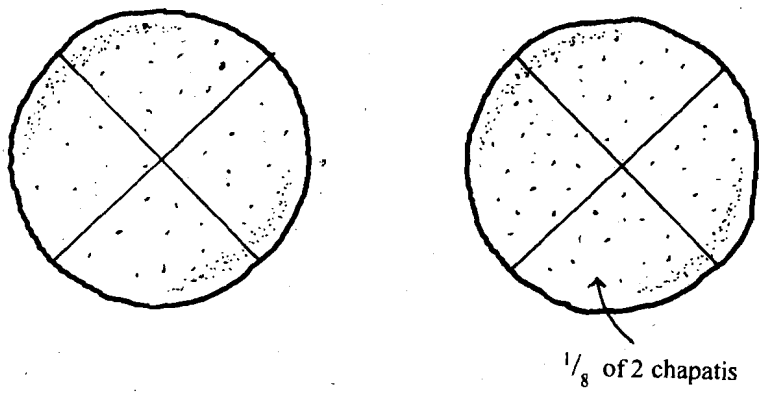

Fig.7

then $\frac{4}{8}$ of 2 chapatis is 1 whole chapati. That means $\frac{2}{4}$ of 2 chapatis is the same as $\frac{4}{8}$ of 2 chapatis.

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Such activities may help the children know two-fourths to be indisputably equivalent to four-eighths or six-twelfths etc. whatever the whole may be.

E4) Do you agree that such activities help children to have a better understanding of the concept of equivalence? If so, why? If not, what would you suggest?

Have you ever asked children to write fractions equivalent to a given fraction? I have, and find that they usually adopt the multiplication method i.e. multiplying numerator and denominator by the same number. They find it difficult to obtain an equivalent fraction by dividing by a common factor. Many people I have asked agree

with this observation. They also say that it is probably because of this that children have problems in reducing a fraction to its lowest terms. In this regard many of you would have had the kind of experience I had with my niece Ankita, a fourth standard student when we were discussing fractions, the other day.

Example 5

I : Can you give me some fraction equivalent to $\frac{8}{12}$?

Ankita : Yes.

Then she tried to remember as I noticed her talking to herself.

$$\frac{8\times2}{12\times2}=\frac{16}{24}$$

$$\frac{8\times3}{12\times3}=\frac{24}{36}$$

$$\frac{8\times4}{12\times4}=\frac{32}{48}$$

Ankita : What did teacher tell me to do here? Yes, now I remember. Multiplying by 2, I get $\frac{16}{24}$. Multiplying by 3, I get $\frac{24}{36}$. Multiplying by 4, I get $\frac{32}{48}$ and like this I can write many more fractions equivalent to $\frac{8}{12}$.

I : Fine! You are right. Now, tell me, is $\frac{2}{3}$ also equivalent to $\frac{8}{12}$?

Ankita : Let me check Auntie.

As I watched her I could notice that she was again trying to check it by the multiplication method. She multiplied $\frac{2}{3}$ (both numerator and denominator) by 2, it did not work. She multiplied it by 3, it did not work. She multiplied it by 4 and shouted.

Ankita : Yes Auntie, even $\frac{2}{3}$ is equivalent to $\frac{8}{12}$. We multiply 2 by 4, and 3 by 4, and get $\frac{8}{12}$.

$$\frac{2\times4}{3\times4}=\frac{8}{12}$$

I : Why don't you do it the easier way? You know 4 is a common factor of 8 and 12. So divide 8 by 4 and 12 by 4 and we get $\frac{2}{3}$.

Ankita : Oh..... it looks easy, but can we do it this way?

I : Why not? Let me show you something. Look at this paper. I fold it so that I get these two equal parts (as in Fig.8). Tell me, what fraction is each part?

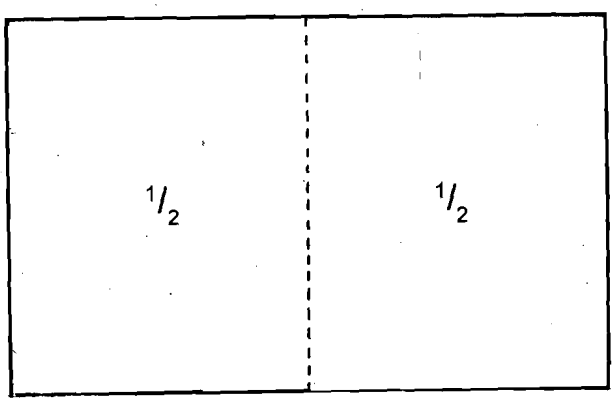

Fig.8

Ankita : $\frac{1}{2}$.

I : Again fold it so that there are 4 equal parts (see Fig.9). Now how many parts is the paper divided into?

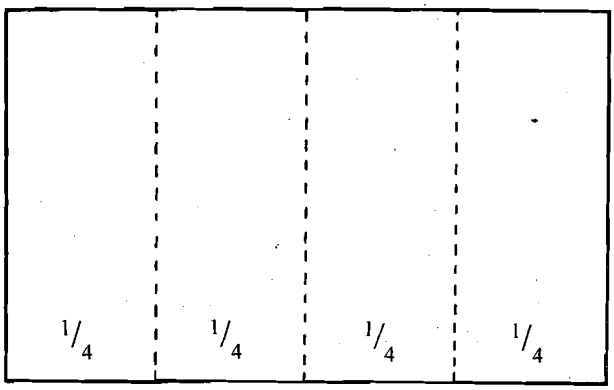

Fig.9

Ankita : Four parts. Each part is $\frac{1}{4}$.

I : Let me open the folds. Do you agree that $\frac{1}{2}$ the paper is the same as 2 one-fourth parts?

Ankita : Yes.

I : So, I can write $\frac{1}{2} = \frac{2}{4}$. Now if I fold this paper once again and then unfold it we get something like this (as in Fig.10).

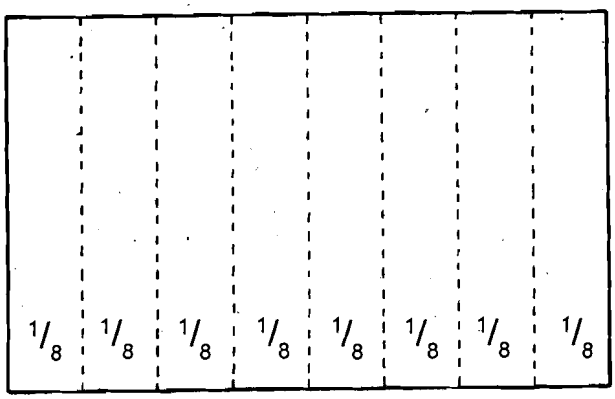

Fig.10

Tell me how many one-eighth parts are there in $\frac{1}{2}$ part of the paper?

Ankita : One two ..... four. Four.

I : So $\frac{4}{8} = \frac{1}{2}$

Ankita : Yes

I : Now can I say that $\frac{4}{8}$ is equivalent to $\frac{2}{4}$ and $\frac{1}{2}$ and write

$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$?

Ankita : Yes, we can do that.

I : That means we can write fractions equivalent to $\frac{4}{8}$ having lower

numerator and denominator. Do you see some pattern when we write $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$?

Ankita : Um, yes. Each time we are dividing the numerator and denominator by 2 as we go from left to right.

I : Does that mean that we have to go step by step and cannot directly write $\frac{4}{8} = \frac{1}{2}$?

Ankita : Why not! If we divide both 4 and 8 straightaway by 4 we get $\frac{1}{2}$.

I : So, do you agree that **we can also write equivalent fractions by dividing the numerator and denominator by their common factor?**

Ankita : Yes Auntie.

I felt happy as Ankita looked convinced and enjoyed doing the activity with me. Next I asked her to do a few exercises like filling up the boxes in $\frac{5}{10} = \frac{\square}{2}, \frac{14}{56} = \frac{2}{\square}$. Ankita took no time to do them correctly. I also utilised this opportunity to explain to her that when we write $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$ then $\frac{1}{2}$ is in the lowest terms because now the numerator and denominator do not have any common factor. So we cannot divide them any further.

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Now here is a related exercise for you.

E5) What method would you use to assess whether the concept of finding equivalent fractions by the division method has got across to your students or not?

You may have noticed that at times children ask why they cannot write equivalent fractions by adding or subtracting the same number from the numerator and the denominator, like we do with multiplication and division? Children should be left to themselves to find out the answers to such problems. Ask them to write a fraction say $\frac{1}{2}$. Let them add 2 to both the numerator and denominator and check if $\frac{1}{2} = \frac{1+2}{2+2} = \frac{3}{4}$.

E6) What would you do to make children understand that $\frac{5}{6}$ is not equal to $\frac{5-2}{6-2} = \frac{3}{4}$.

Yet another problem children usually face is to understand the equivalence of $\frac{4}{4}, \frac{5}{5}$ ... to 1 and write $\frac{4}{4} = 1$, or $\frac{5}{5} = 1$. The following scene may help you to make your students understand it.

Example 6

Teacher : Here is an apple. I cut it into four equal parts. So these four parts put together is a whole apple. Now Renu, come here and take this one part.

Renu : Yes madam.

Teacher : What portion of apple have you got Renu?

Renu : One part.

Teacher : Out of how many parts?

Renu : One out of four parts.

Teacher : Write it on the board

Renu : $\frac{1}{4}$.

In the same way the teacher gave Renu a second piece and then a third piece and made her write $\frac{2}{4}$ and $\frac{3}{4}$ on the board i.e. the fraction for the amount of apple she got. Finally, she gave the last piece to Renu.

Teacher : Now how many parts of the apple have you got?

Renu : Four parts.

Teacher : Out of how many parts have you got 4 parts?

Renu : 4 out of 4.

Teacher : So can I write the fraction for this as $\frac{4}{4}$.

Renu : Yes madam.

Teacher : O.K. children tell me how much of the apple has Renu got till now?

Students : Full/Whole/One.

Teacher : Can I write $\frac{4}{4} = 1$

Students : Yes madam.

Teacher : In the same way we have $\frac{5}{5} = 1$, $\frac{6}{6} = 1$ ... etc.

The above activity can also be done with a piece of paper cut into four equal pieces or with 4 dice or 4 marbles.

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Once the children understand that $\frac{4}{4} = 1$, $\frac{5}{5} = 1$ .... etc. It will help them to understand that 5 one-fourth parts $\left(\frac{5}{4}\right)$ is a fraction greater than 1. If a sheet of paper is divided into 4 parts then 5 one-fourth parts would cover one whole sheet and would require the use of another sheet, and therefore is a fraction greater than one.

E7) Suggest an activity for explaining $\frac{5}{4}$ to a Class IV child.

You would agree with that a child needs to understand how to order fractions before we teach her subtraction We shall talk about it now.

13.4 ORDERING FRACTIONS

A child's understanding of the ordering of two fractions that is deciding which of the relations 'is equal to', 'is less than', or 'is greater than' holds for two fractions, requires an understanding of the ordering of unit fractions. This can be done by doing a simple activity like the following.

Fractions with numerator one are called unit fractions

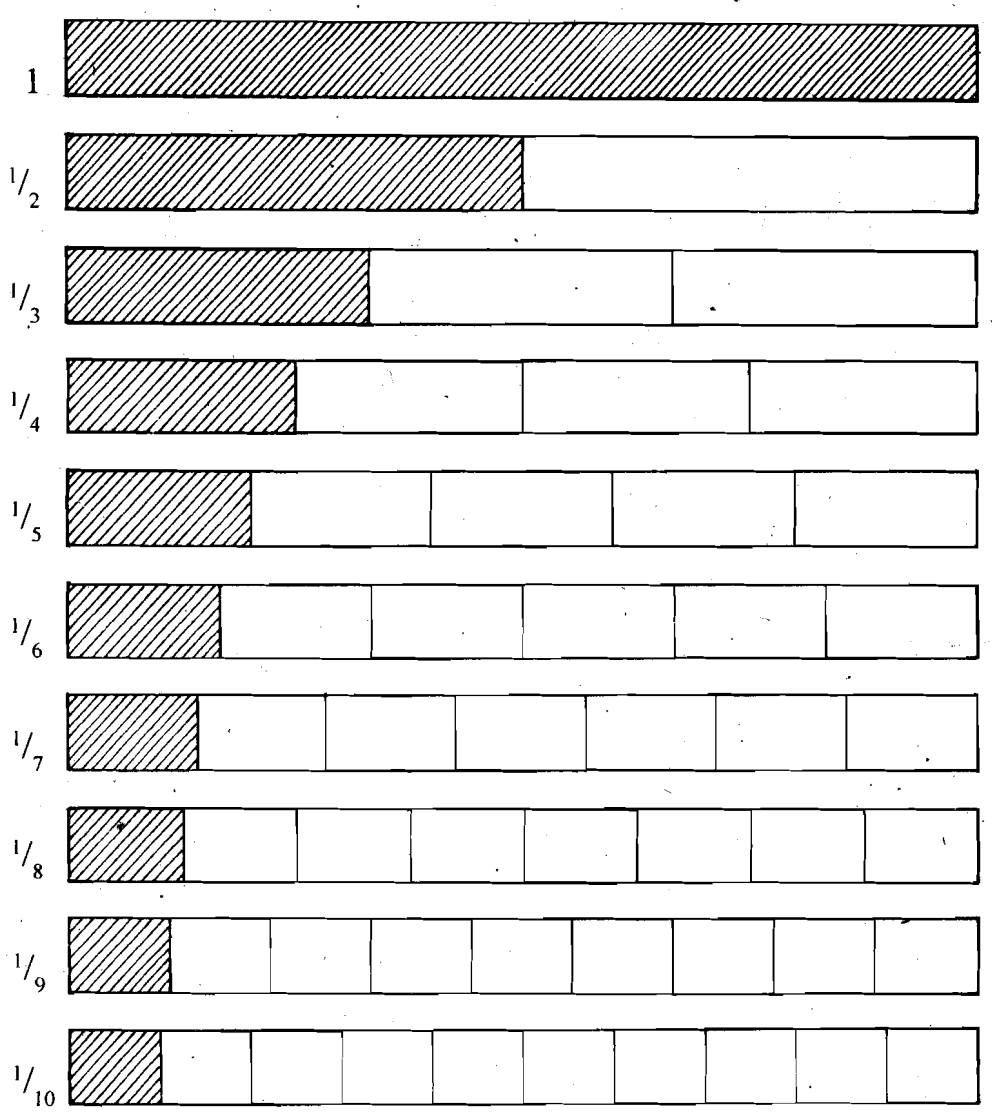

Fig.11

Example 7: Take ten strips of paper which are equal in size. Keep the first strip as it is. Divide the second strip into two equal parts, third into three equal parts,... and like this the tenth into ten equal parts. Then paste them on a card board one below the other. Shade one part of each strip. (see Fig.11)

Showing the children the different shaded portions of the same sized strips helps them to realise that $1 > \frac{1}{2} > \frac{1}{3} > .... > \frac{1}{10}$.

This activity for showing $1 > \frac{1}{2} > \frac{1}{3} > .... > \frac{1}{10}$ can also be tried using the number line. It may act as a visual aid for the learners for representing fractions. You can test it by trying it with your students.

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E8) Using a number line show that $1 > \frac{1}{2} > \frac{1}{3} > .... > \frac{1}{10}$ in your class. How do your students react to it? Do they find it difficult to grasp the representation of fractions on a number line or does it turn out to be an effective visual aid for them?

Understanding the important relationship between the size and number of parts into which a whole is divided is very important in determining the order of fractions. If a child has a clear understanding about it then it becomes easier for him/her to order unit fractions, fractions with the same denominator and fractions with the same numerator. But sometimes children interpret this relationship between the size and number of parts wrongly. This reminds me of the experience I had with my 8-year old niece Ankita.

Once, I was talking to her about ordering of fractions.

Ankita looked very confident about ordering unit fractions. She took no time in saying that $\frac{1}{8}$ is less than $\frac{1}{5}$. She explained the way she understood it that if she shares a cake with 7 other children her share would be less than what she would get if she shares it with 4 other children. She could very easily order fractions like $\frac{1}{3}, \frac{1}{6}, \frac{1}{11}, \frac{1}{20}$. But I was surprised when she said $\frac{7}{8}$ is less than $\frac{4}{8}$ because seven is more parts and as the parts become more in number they become smaller in size. I said let's see. I took two paper strips of the same size and divided each into 8 equal parts (see Fig.12). Ankita helped me in doing this.

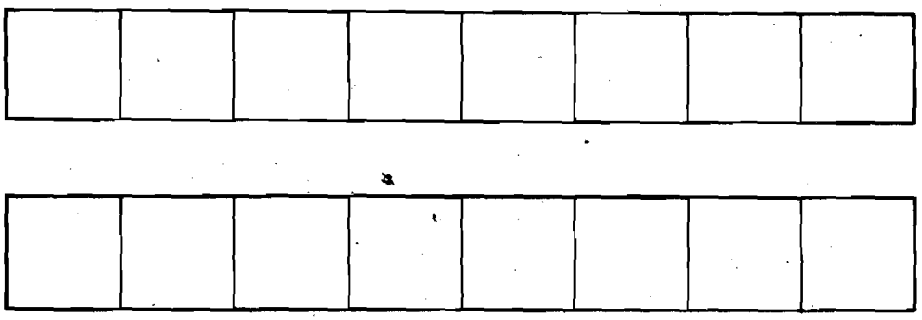

Fig.12

Example 8

I : Ankita, now take the first strip and colour 4 parts out of 8 red.

Ankita : O.K. Auntie. Here it is (Fig.13)

Fig.13

I : What fraction is this red part?

Ankita : $\frac{4}{8}$.

I : O.K! Now take the second strip and colour 7 parts out of 8 grey.

Ankita : Here it is (Fig.14)

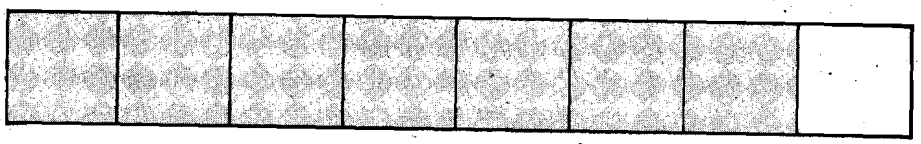

Fig.14

I : What fraction is this grey part?

Ankita : $\frac{7}{8}$.

I : Which is more the red shaded part or the grey shaded part?

Ankita : Grey is more

And before I could ask her any other question she herself came out with the answer. By now she was enjoying herself and wanted me to do some more activities of this kind. So I showed her $\frac{4}{4}$ and $\frac{5}{4}$ with the help of a paper. In between I asked her a few questions like which of the two $\frac{4}{4}$ and $\frac{5}{4}$ is more? Why is it so? and she responded well.

Now she was very clear that $\frac{4}{13}$ is less than $\frac{4}{8}$. But to decide which of $\frac{7}{8}$ or $\frac{12}{13}$ is less was still a problem for her. I thought I must get the answer from her and proceeded as follows:

I : Ankita, do you have any problem in comparing numbers like $\frac{7}{8}$ and $\frac{11}{8}$, or $\frac{3}{7}$ and $\frac{4}{7}$ etc.?

Ankita : No auntie, it's very simple. I don't see any problem here.

I : Why do you think it's easier to compare $\frac{7}{8}$ and $\frac{11}{8}$ than comparing $\frac{7}{8}$ and $\frac{12}{13}$?

Ankita : Here it's easier because the numbers have common denominator and so we have to only bother about the numerators.

I : Fine! But can't you change $\frac{7}{8}$ and $\frac{12}{13}$ to equivalent fractions with the same denominator?

After little bit of thinking

Ankita : Yes I can do that.

She did some calculations and came out with the answer.

Ankita : The two numbers equivalent to $\frac{7}{8}$ and $\frac{12}{13}$ are $\frac{91}{104}$ and $\frac{96}{104}$.

I : Can you decide now which is greater $\frac{7}{8}$ or $\frac{12}{13}$?

Ankita : (With a smile on her face) Yes $\frac{12}{13}$ is bigger.

Ankita looked thrilled and happy. She really enjoyed doing such an exercise and looked confident. I also felt happy,

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Once you feel that children have clearly understood the concept of ordering fractions through experience with physical and diagrammatic models they could be introduced to a general algorithm—usually called a **common denominator algorithm** to determine the order of fractions. According to this **for comparing two or more fractions change them to equivalent fractions with the same denominator. Then the fraction with the greatest numerator is the largest.**

E9) Which is bigger $\frac{12}{13}$ or $\frac{7}{8}$? How will you make a child understand this without applying 'a common denominator algorithm'.

We will now talk about a common problem that children usually face, namely, converting improper fractions to mixed fractions.

13.5 IMPROPER FRACTIONS

Fractions with numerator greater than denominator are called improper fractions.

Mixed fractions are sum of whole numbers and a proper fraction.

Primary school teachers usually complain that children while dividing an improper fraction to convert it into a mixed fraction usually write the remainder instead of quotient as the whole number part of the mixed fraction. I happened to notice the same thing last Sunday when I visited a friend's house. Her 9 year old daughter Mini was busy doing her homework. I just glanced at her note book. She was doing sums on fractions converting improper fractions to mixed fractions. She divided 7 by 4 as

$$4\overline{)\,7}\;\;1$$
$$\underline{4}$$
$$3$$

and wrote $\frac{7}{4} = 3\frac{1}{4}$. I quietly watched

her. She again wrote $\frac{11}{6} = 6\overline{)\,11}\;\;1 = 5\frac{1}{6}$.
$$\underline{6}$$
$$5$$

While talking to her it appeared that she had some vague idea of what her teacher might have told her about the conversion of improper fractions into mixed fractions and was doing sums mechanically without having in mind any visual picture of what she was doing. I tried to make it clear to her and proceeded as follows

Example 9

I took 5 circular paper cuttings each divided into four equal parts (see Fig.15)

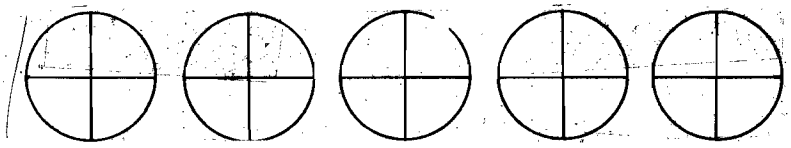
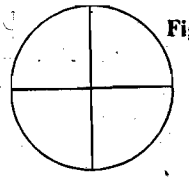


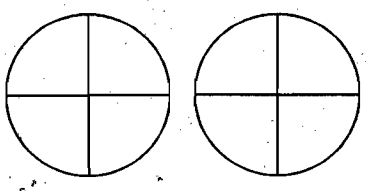
Fig.15

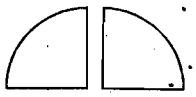
I : What is $\frac{1}{4}$ of 

Mini : It is one part out of 4.

I : Is it ?

Mini : Yes

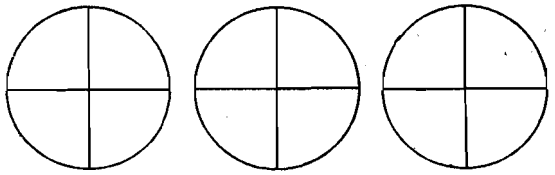
I : O.K.! Now tell me what is $\frac{1}{4}$ of 

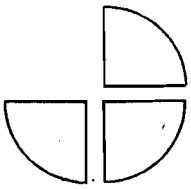
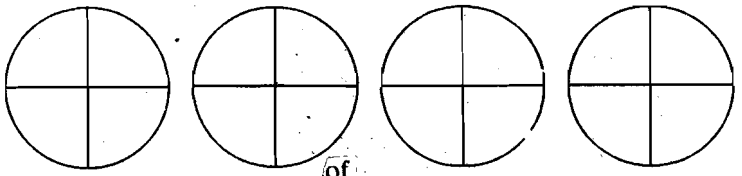
Mini : It's  .

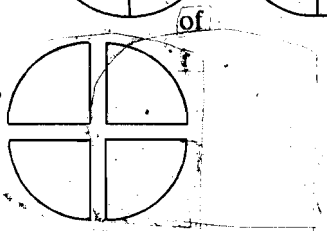
I : What fraction does it represent?

Mini : It's 2/4.

I : Good. So it's two one-fourths. In the same way can we say

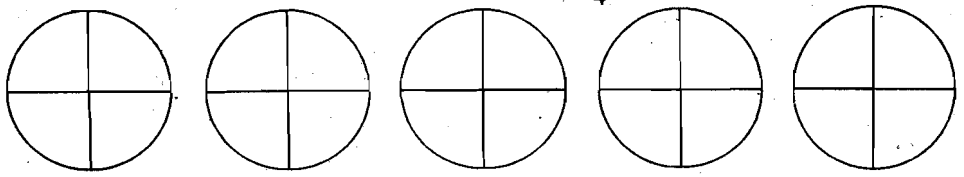
$\frac{1}{4}$ of  i.e. three one-fourths

 $= \frac{3}{4}$ and $\frac{1}{4}$ of 

or four one-fourth is 

Mini : Yes auntie.

I : **O.K.**! Now can you tell me what is $\frac{1}{4}$,

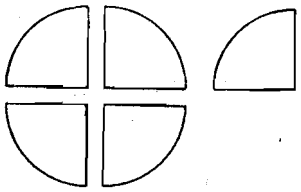


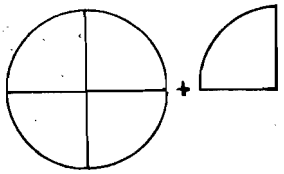
or five one-fourths

Mini looked a bit confused so I thought of helping her

I : Mini isn't $\frac{5}{4}$ same as $\frac{1}{4}$ of 5.

Mini : Amm ... Oh Yes! Now I can tell you what $\frac{1}{4}$ of 5 is. It's



I : Isn't it same as  $= 1 + \frac{1}{4}$.

Mini : Yes auntie they are same.

I : O.K. This means that $\frac{5}{4}$ is 1 whole and $\frac{1}{4}$ more. We write it together as $1\frac{1}{4}$.

Now when we divide 5 by 4 as $4\overline{)5}$ $\frac{4}{1}$ then the quotient gives the whole number part of the mixed fraction and $\frac{\text{remainder}}{\text{divisor}}$ gives the proper fraction part of it. So we have $\frac{5}{4}$ = Quotient $\frac{\text{remainder}}{\text{divisor}} = 1\frac{1}{4}$.

Now what do you have to say about your number $\frac{7}{4}$? Mini just looked at her notebook and smiled.

Mini : Now I am clear about it. According to what you have just told me $\frac{7}{4}$ is 1 whole and $\frac{3}{4}$ more. So it's $1\frac{3}{4}$.

Mini understood her mistake. She looked very confident as she answered a few more problems of the same type correctly.

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And now here is an exercise for you.

E10) What was Mini's problem?

E11) Could you suggest any other method which you might have followed to correct Mini's mistake?

13.6 SUMMARY

In this unit we have tried to bring the following points.

1) It is usually noticed that children find it difficult to name the fractions and symbolise them. Understanding the notation by which fractions are symbolised is a problem for them. Any confusion in a child's mind about the procedure by which a fraction is named should be cleared before proceeding with the topic. It is only after children become familiar with fraction names such as a 'half', 'third', 'quarter' etc. and the meaning of these names in relation to various wholes they should be introduced to symbols for fractions. It should be clear to them that when we draw a line under '2' and write '3' beneath the line as $\frac{2}{3}$ then the denominator of the fraction $\frac{2}{3}$ tells us that the whole has been divided into three equal parts and the numerator tells us that we are looking at two of these parts. They should be given a lot of practice in translating fraction names to fraction symbols. Various activities can be taken up in the class by the teacher in this regard.

2) The numerator-denominator notation for fraction also makes it possible to denote the same fraction in infinitely many ways. As a group activity children can be asked to find as many numbers as they can for a particular fraction. After they have done such activities they may be introduced the concept of equivalent fractions. Paper folding activity and other similar activities for different categories of whole help children understand why more numbers are equivalent. Once the children have got enough experience of finding the fractions equivalent to a given fraction let them observe the pattern followed in each case and discover the rule for writing equivalent fractions.

3) Once children become familiar with the rule for writing equivalent fraction enough practice should be given to them for obtaining an equivalent fraction by dividing the numerator and the denominator by a common factor as it has been observed that children find it difficult to apply.

4) Sometimes children interpret the relationship between the size and number of parts into which a whole is divided wrongly. This also affects their understanding of ordering fractions. Paper shading activity can help them clear their doubts about deciding which number is greater. Teacher can start with comparing unit fractions and then go to fractions with common numerator or denominator. This way they can themselves evolve the pattern

for comparing other fractions like $\frac{5}{6}$ and $\frac{7}{8}$, $\frac{3}{4}$ and $\frac{6}{7}$ etc. by generating a general algorithm called a common denominator algorithm.

5) Problem of converting an improper fraction into mixed fraction which commonly occurs among the children aged 8-10 years can be taken care by doing activities with the concrete objects and actually showing what $\frac{7}{4}$ (or, seven one-fourths or, $\frac{1}{4}$ of 7) means. This will give them visual picture of what they are doing and help them in understanding, the concept.

13.7 COMMENTS ON EXERCISES

E1) The illustration in Fig.3 and the explanation given by Ankita is correct. Children need a great deal of time and discussion to focus on the correct aspect. Many examples of naming fractions are required to develop a sense of labelling fractions. A teacher in a class may divide the children into groups and each group be asked to record each fraction of the whole piece of paper in as many different ways as possible. Each different way of recording it earns them an extra point. A team with maximum points is the winner. Such activities will help them build up experience of naming fractions without their knowing that they have gained background experience of the equivalence of fractions.

E2) In this regard once I happened to see a teacher doing an activity in the class which I would like to share with you. Teacher divided the children in a class into 4-5 groups. Then she drew a figure on the board (See Fig. 16).

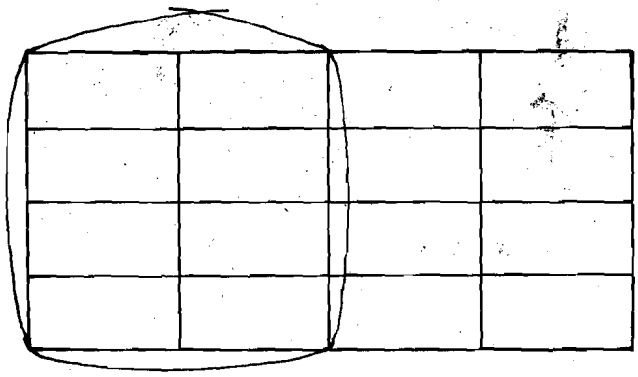

Fig.16

She wanted children to draw it on the paper and tell everything they can about the ringed area. Children worked in groups and recorded their results. They also explained to one another why it had been given that particular name. It is called a 'half' because there are two pieces like it on the whole sheet. It is called 'four-eighths' because two rectangles are called 'one-eighth' and there are four pieces of that size. Like this the teacher made them give reason for every answer they had written like it's two-quarters or eight-sixteenths or four-sixteenths etc. Children wrote out the names in word as well as in standard fraction notation because each different name and each different way of recording it gave them an extra point. Through this activity teacher could judge the amount of experience children had about the fraction-naming concept.

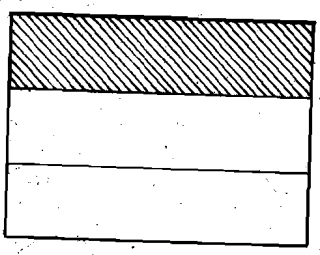

Fig. 17

E3) For showing fractions equivalent to $\frac{1}{3}$ take a rectangular paper and fold it into 3 equal parts. Press it along the sides and then unfold it. Paper is divided into 3 equal parts (see Fig.17). Fold it once again and then unfold it. It is divided into 6 equal parts (see Fig.18). Repeat the process once again the paper will be divided into 12 equal parts as shown in Fig.19. Now by

colouring $\frac{1}{3}$ of Fig.17, $\frac{2}{6}$ of Fig.18, and $\frac{4}{12}$ of Fig.19, you can show that $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$.

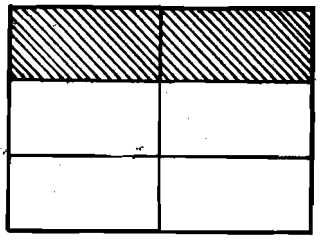

Fig. 18

Again using Fig.17, 18 and 19 you can show that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$ and $\frac{8}{12}$.

Fractions equivalent to $\frac{1}{4}$ can also be shown by the paper folding activity. This time you have to start with folding the paper into 4 equal parts.

E4) To find out the utility of having such activities, I talked to few teacher. Some of them were of the opinion that such activities do help children to have a better understanding of the concept of equivalence. They feel that these activities help them see and realise why $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ is equivalent to $\frac{4}{12}$. It help them eliminate all the errors in their thinking and fill up the gaps in their understanding. Specialisation over a large number of cases help children in the identification of an underlying pattern. But you may not agree with them, so you can try these activities in your class and write your views about it.

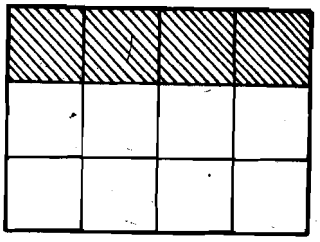

Fig. 19

E5) One way of assessing is to do exercises in the class where you can ask children to write fractions equivalent to a given fraction with lower numerator and denominator. Then going around you can check their responses. In between you can ask them to check their solution by paper folding also.

E6) This can be done by paper folding or shading. Ask children to take two sheets of paper of the same size. Let them divide one into 6 equal parts and another into 4 equal parts. Ask them to shade 5 parts out of 6 of the first sheet green, and 3 parts out of 4 of the second sheet red. Now let them compare the two coloured regions and decide whether $\frac{5}{6}$ is equal to $\frac{3}{4}$ or not. This exercise can also be done by taking two sets of 12 marbles each. Divide one set into 6 groups of 2 each and see how many marbles are there in 5 such groups. Then divide second set of 12 marbles into 4 groups of 3 each and see how many marbles are there in 3 such groups. You can now easily show that $\frac{5}{6}$ is not equal to $\frac{3}{4}$.

E7) Simple activity of paper folding can be done to show $\frac{5}{4}$. Various steps involved are

Step 1 : Fold one region into 4 parts.

Step 2 : Point to a region that is $\frac{1}{4}$ of the whole. Can you shade 5 one-fourth parts? No! We need to fold another region in order to show 5 one-fourth parts.

Step 3 : Fold another region. Cut 1 part out of four from this region and paste it along the first region (the whole of it). Now you can show $\frac{5}{4}$.

E8) While showing that $1 > \frac{1}{2} > \frac{1}{3} > \ldots\ldots > \frac{1}{10}$ on a number line be careful that each time you draw a number line and take a unit length on it, this length should be same. Also when further dividing this unit length into required

number of intervals they should be of equal length so that children realise that $\frac{1}{8} < \frac{1}{7}$ or $\frac{1}{7} < \frac{1}{6}$ etc. Some teachers feel that using the number line to integrate the concept of a fraction with the concept of a number is a good and effective method of teaching.

E9) $\frac{12}{13}$ is bigger than $\frac{7}{8}$ can also be shown using a number line.

You can show (see Fig.20) that

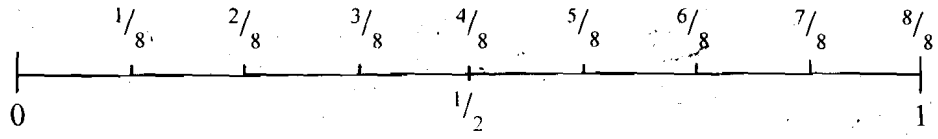

Fig.20

$\frac{8}{8}$ is 1 whole and $\frac{7}{8}$ is 1 part i.e. $\frac{1}{8}$ away (less than) from 1 whole. Similarly $\frac{13}{13}$ is whole and $\frac{12}{13}$ is 1 part i.e. $\frac{1}{13}$ away from 1 whole (see Fig. 21)

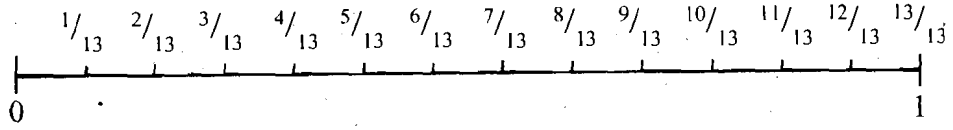

Fig.21

Now which is smaller $\frac{1}{13}$ or $\frac{1}{8}$? Let the child come out with the answer and then decide which number is bigger $\frac{12}{13}$ or $\frac{7}{8}$. If a child finds it difficult then you can help him/her by asking a few more questions in between.

E10) It appeared that Mini never got in opportunity to experience what $\frac{7}{4}$ is with the help of concrete objects, or paper shading/folding activity. She only knew these numbers $\frac{7}{4}$ or, $\frac{11}{6}$ as improper fractions, which were to be changed to mixed fraction notation and for which her teacher had told her some method.

E11) In order to correct Mini's mistake it was necessary to make her realise that $\frac{7}{4}$ means taking away 7 parts out of 4 parts and for that we require 1 full whole and another whole from which we need to take away 3 parts out of 4. She should be made to realise that $\frac{7}{4}$ = 1 whole and $\frac{3}{4}$ more. We can do this by doing a paper shading activity also. We can show that

$$\frac{7}{4} = 1 + \frac{3}{4} = 1\frac{3}{4} = \text{Quotient}\frac{\text{remainder}}{\text{divisor}}$$

UNIT 14 OPERATIONS WITH FRACTIONS

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14.1 INTRODUCTION

From the earlier units and from your own experience you must be aware of the various errors that children make while dealing with fractions. But how many of us have looked into the possible reasons for such errors?

In Units 8 and 9 we have already discussed ways in which children can be helped to overcome the common errors which they make while performing the four mathematical operations with whole numbers. As you will see in this unit the errors and misunderstandings in performing these four operations with fractions are not the same as those related to whole numbers. And therefore, we need slightly different teaching strategies for helping the children to deal with them. Of course, they need to have a good basis in the four operations with whole numbers to be able to have a clear understanding of the operations with fractions. We also talk about other concepts and skills which children must master to perform these operations successfully.

Throughout this unit, we have discussed a variety of teaching aids that can be used to help children perform the operations correctly. We hope that this unit will help you to think of many more.

Objectives

After studying this unit, you should be able to

- use various activity-based methods to enable children to learn how to add, subtract, multiply and divide fractions,
- help children to establish the rules for combining fractions;
- develop methods for continuously evaluating the effectiveness of your teaching strategy.

14.2 DEVELOPING UNDERSTANDING IN ADDITION AND SUBTRACTION

"I have told them so many times how to add two fractions. And yet, they make mistakes while finding $\frac{2}{3} + \frac{4}{8}$. Some have written it as $\frac{6}{8}$, some as $\frac{6}{11}$....". This was a remark from a frustrated mathematics teacher whom I happened to visit one day. She was correcting the answer scripts of Class IV students. Have you heard similar remarks from other teachers? Can you recall an occasion when you have felt similarly frustrated?

In this case, she seemed to be frustrated because she had spent a lot of energy explaining the same thing again and again, to no effect. But, did she stop to analyse

what might be wrong? Why were her students not able to understand the underlying principle in addition of fractions?

I thought I'd provoke her to think over these matters. So, I asked her the following questions

- Do you know what notions children have regarding addition and fractions?
- Have you noticed a pattern in their responses, and the possible reasons for the pattern?
- Can you think of other kind of activities that deal with similar ideas and allow them to get around the bottle necks in their understanding?

She thought over these questions. Some time later she said, "Yes, I suppose I didn't think about other ways which can make children deal with this concept. I just reproduced the same explanation over and over again." She decided to talk to her colleagues about alternative ways of getting children to practice and learn. What she realised was that

Repeatedly giving a child the same explanation is not the best way to help her understand a concept.

Let us now look at some examples of methods for teaching how to add and subtract fractions. We shall then discuss how effective these methods are.

Example 1: To start with, let us consider an experience of a friend of mine, who teaches in a primary school. She was taking a remedial session for some Class 4 children who were making common mistakes like $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$. After interacting with the children, she realised that the problem arose because they had been forced to merely memorise the 'rule of addition' and solve the problems, without understanding how the rule worked. So she decided to use some simple aids to give the children some concrete illustrations of the rule.

She started with trying to help them grasp what $\frac{1}{3} + \frac{1}{3}$ is. She gave all the children sheets of paper. Then she took a sheet of paper, folded it into three parts and unfolded it (as in Fig.1) and asked the children to do the same.

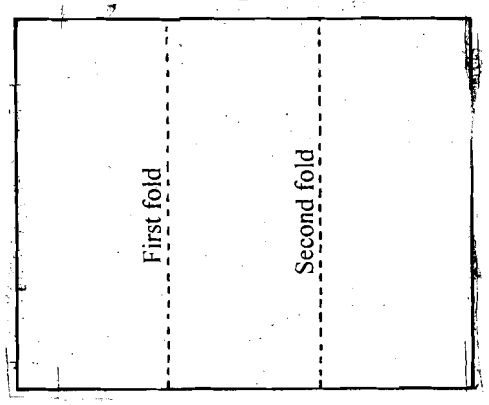

Fig.1: A sheet of paper divided into three parts.

Teacher : Now tell me what fraction of the sheet of paper does each portion represent?

Children : $\frac{1}{3}$

Teacher : Now, shade (by filling the oblique lines) the first fold and mark $\frac{1}{3}$.

on it. Then shade the next portion and write $\frac{1}{3}$ on that (see Fig.2). (They do so.). Now how many parts are there in the shaded portion?

Children : Two

Teacher : Two out of three. So, what fraction is represented by the shaded portion?

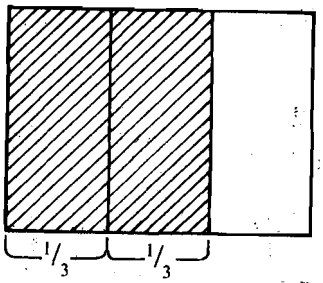

Fig.2: Shaded portion shows $\frac{2}{3}$

Children : $\frac{2}{3}$

Teacher : So can we write $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$?

Children : Yes, madam.

Teacher : Now let us add $\frac{3}{4}$ and $\frac{1}{4}$. Take another paper. How many parts should we fold the paper into?

One Child : Four parts.

Teacher : O.K., fold it into four parts. Now first shade (by filling with lines)3 parts of it, then fill the remaining part with dots (see Fig.3). So what fraction is represented by the shaded portion?

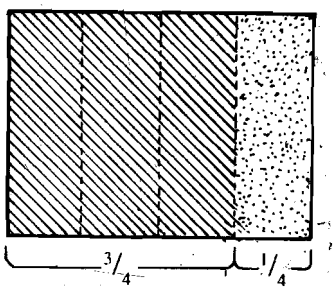

Fig.3

Children : 3 by 4. (Teacher writes $\frac{3}{4}$ on the board).

Teacher : And the portion with dots?

Children : 1 by 4. (Teacher writes 1/4, next to 3/4)

Teacher : So, what portion of the paper is covered by your pencil marks?

Children : Whole/full/1 page.

Teacher : Does it mean that the sum of $\frac{3}{4}$ and $\frac{1}{4}$ is 1?

Children : Yes, madam.

Teacher : Can I write $\frac{3}{4} + \frac{1}{4} = 1$?

Children : Yes, madam.

Then the teacher gave the children some more problems of this kind to solve. Next day she decided to use the same method to deal with a slightly more complicated problem like $\frac{3}{5} + \frac{3}{5}$. This time she asked the children how to proceed.

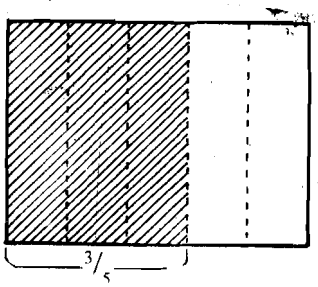

Fig.4

One Child : Fold the paper into 5 parts. (After some time) $\frac{3}{5}$ means, 3 parts out of 5 parts. So first we shade (by filling with lines) 3 parts out of 5 (see Fig.4). $\frac{3}{5} + \frac{3}{5}$ means, we have to shade 3 more parts out of the remaining parts of the sheet. (Child, a little unsure). But there are only 2 parts left. What to do then?

Teacher : (The teacher shaded by filling with horizontal lines the remaining two portion) O.K., what does that show? (Silence in the class.)

Can we use another paper?

Another Child : We can use another paper folded into 5 parts out of which we take 1 part to make it a total of 3 parts like this (see Fig.5).

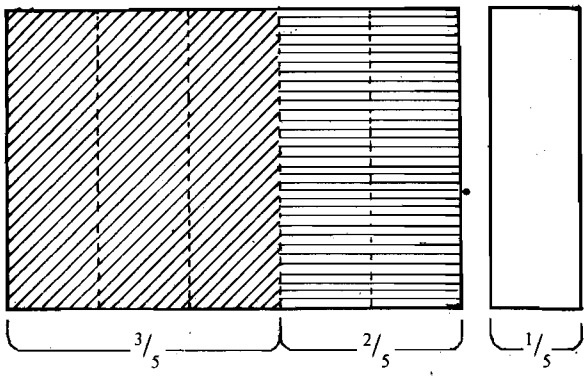

Fig.5: Visualising 3/5 + 3/5.

The teacher took another sheet, folded that into 5 parts and cut out one portion from that and shaded it (by filling with horizontal lines). Then she gave it to the child who held it alongside the paper as shown in Fig.5.

Teacher : But then what can you say about this sum $\frac{3}{5} + \frac{3}{5}$?

The Child : It is more than 1.

Teacher : Good! How much more than 1?

Another Child (after a bit) : One part more

Teacher : One part more out of how many?

Children : Out of five parts

Teacher : Does it mean that the sum is $\frac{1}{5}$ more than 1.

Children : Yes, madam.

Teacher : Then how much is $\frac{3}{5} + \frac{3}{5}$?

One Student : It is $1 + \frac{1}{5}$

Another Student : It is $1\frac{1}{5} = \frac{6}{5}$

Teacher : That's correct. Can I also write $\frac{3}{5} + \frac{3}{5} = \frac{5}{5} + \frac{1}{5} = \frac{6}{5}$?

Children : Yes madam.

Teacher : (Summing up whatever she did in the class) So we have shown that

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}.$$

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$$

$$\frac{3}{5} + \frac{3}{5} = \frac{6}{5}$$

Now, can you say what $\frac{2}{7} + \frac{4}{7}$ is, without folding paper?

One Child : It is $\frac{6}{7}$.

Teacher : Very good. How did you get that?

Child : Add the numbers in the numerator and write the common denominator.

Teacher : Fine. Do you remember what fractions having a common denominator are called?

A few Children : Yes madam. Like fractions.

Teacher : Good! So, to find the sum of two like fractions, you have to add their numerators, and the denominator is the common denominator.

Then the teacher gave a lot of practice exercises.

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Here we make a note of one thing.

Note 1: Be careful while explaining to the child what $\frac{3}{5} + \frac{3}{5}$ is. While explaining, it is better not to keep the two charts alongside as shown in Fig.6.

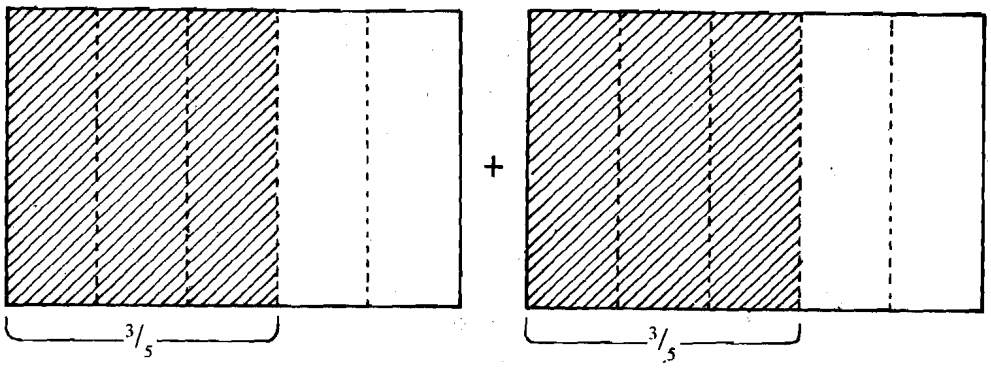


Fig.6

This has been seen to lead the children towards the wrong notion that $\frac{3}{5} + \frac{3}{5} = \frac{6}{10}$, because they count the total number of shaded portions in the two sheets as $5 + 1 = 6$ and the total number of portions into which the two sheets are divided as $5 + 5 = 10$.

Of course, there can be several reasons for a child acquiring misconceptions. Sometimes the child may continue to believe, that 3/5 + 3/5 is 6/10 regardless of the variety of activities that you give her for dispelling this notion. In such a situation what has been found helpful is to "correct" the child, that is give her the fact that $\frac{3}{5} + \frac{3}{5}$ is $\frac{6}{5}$.

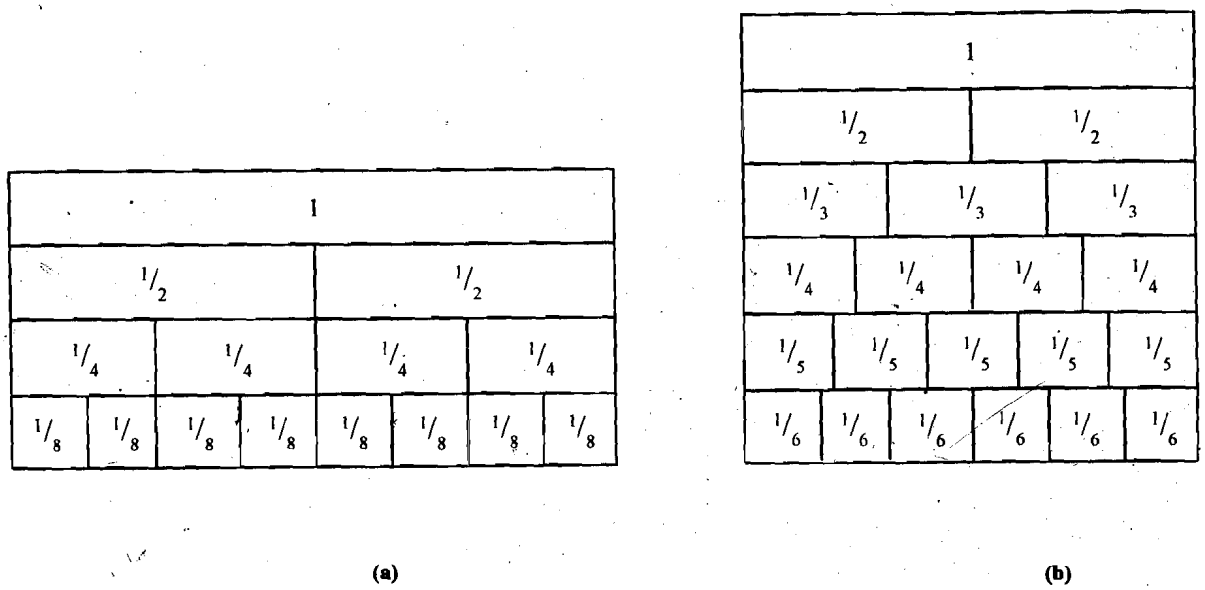
Now try some exercises related to what we have just discussed.

E1 Do you think that the paper folding activity a good activity for teaching addition of fractions? How would you go further, and teach them to subtract like fractions, say for example, finding $\frac{3}{4}-\frac{1}{4}$?

E2 Suggest a group activity for introducing addition/subtraction of fractions.

You would agree with me that paper folding is an effective teaching aid for teaching addition of two like fractions. But it is observed that it can't be used so effectively while teaching the addition of two unlike fractions. So let's see what the teacher does in this situation.

Example 2: In the next class the teacher brought with her a few fraction charts like the one shown in Fig.7. The students were already familiar with these charts as they had used them while learning equivalent fractions (see Unit 13). She placed the charts near the blackboard and started her lesson by checking whether the children had understood "mixed fractions and equivalence of fractions." These are the concepts that a child needs to know before he/she can be expected to deal with operations on fractions. When the teacher was satisfied with the students performance, she started with addition of fractions.



(a) (b)

Fig.7: Fractional Charts.

Teacher : What is $\frac{1}{4}+\frac{1}{4}$?

Children : (unanimously): $\frac{2}{4}$.

A unanimous remark from a class does not necessarily indicate that every child has understood the concept being taught. In fact we should rotate the questions in the class room to evaluate how much each child has understood.

Teacher : What is $\frac{1}{3}+\frac{1}{3}$?

Children : (roared again): It's $\frac{2}{3}$

Teacher : What is $\frac{2}{3}+\frac{1}{3}$?

One Child (After a bit of silence) It's one.

Teacher : Can you explain how?

Raju : (Pointing at the third row of the chart) Madam, each part is $\frac{1}{3}$. First I shade (by filling with lines) the first two parts. I get $\frac{2}{3}$. Now I shade the third part (by filling with different lines) also and I get $\frac{1}{3}$ more. Now the whole strip is shaded, so it's 1.

Teacher : Very good.

After confirming that the others had understood it, she went on.

Teacher : O.K.! Now tell me what is $\frac{1}{3} + \frac{1}{2}$?

There was complete silence in the class. The teacher again pointed at Raju.

Raju : humm..... .

Teacher : What is the problem?

Raju : Madam, if they were on the same strip then it was possible. But, in this case, they are on different strips.

Teacher : O.K., look at the fractions on the same strip (Fig.7) (Pointing at the second row of the chart). Here the whole is divided into two parts and each part shows $\frac{1}{2}$.

The teacher then pointed at the third row, and asked them appropriate questions so that they themselves came out with the information that it was divided into 3 parts, and each part being $\frac{1}{3}$ of the whole.

Teacher : Into how many parts should we divide a whole so that we can show $\frac{1}{3}$ and $\frac{1}{4}$ on the same strip? (After waiting a bit for some child to answer). Let's try with 4 divisions. Now the fourth row on the chart is divided into 4 parts. So can we show $\frac{1}{2}$ on this?

One Girl : Yes madam, 2 out of 4 parts (2/4) is the same as $\frac{1}{2}$.

Teacher : Now can we show $\frac{1}{3}$ on this strip?

One boy : No madam, we can't.

Teacher : Then let's try with the fifth row.

Children (Entire class roared) No madam, in the fifth row we can neither show $\frac{1}{2}$ nor can we show $\frac{1}{3}$.

Teacher : What about the sixth row with 6 divisions?

One boy : Yes madam, it's possible with this row. Each part is $\frac{1}{6}$. So if we take 2 out of 6 parts then they are the same as $\frac{1}{3}$, and if we take 3 parts out of 6 we get $\frac{1}{2}$.

Teacher : Very good.

She then drew one strip on the board divided it into 6 parts,and called one student to first shade 2 parts out of 6 and then 3 more parts out of the remaining parts, as in Fig.8.

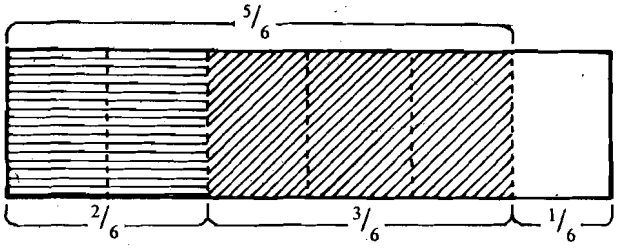


Fig.8: Visualising $\frac{1}{3} + \frac{1}{2}$.

Teacher : Tell me how many parts are shaded now?

Children : 5 out of six.

Teacher : So, what is $\frac{1}{2} + \frac{1}{3}$?

A few Children. : $\frac{5}{6}$

Teacher : So is $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$?

Children : Yes madam,

The teacher did a few more exercises like $\frac{1}{4} + \frac{1}{3}$ with the children. Soon they looked more and more confident about addition of fractions.

Here's an exercise now.

E3 Do you feel that the activity discussed in Example 2 is effective? What are its advantages and disadvantages?

The teacher observed that each time the children reasoned out their calculations for a problem their thinking got more and more fluent and they started recognising the pattern involved in the addition of fractions. Though they were not given the rule they naturally arrived at the rule for addition

Note: Of course, as we know the general rule is that **to add fractions with the same denominator add the numerators and keep the same denominator**

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a \times d + c \times b}{bd}$$

This general rule is too abstract for children to grasp in this form. Though as you have seen in the example above, they can grasp the rule in the particular case.

Summarising whatever we have discussed so far, we conclude that the teaching of addition of fractions involves three stages, namely

(1) children should be given considerable time for understanding the concept of fraction equivalence. This concept must be understood clearly before other tasks are performed.

(2) the idea of adding fractions with the same denominator should be introduced first. It should be developed through experience with physical and diagrammatic models which help them to observe the common pattern.

(3) Addition of unlike fractions.

If the children go through the first two stages, they realise why we need the same denominators to add fractions, and hence why we replace the fractions by their equivalent ones with common denominators. Let me quote an instance here.

A teacher who helped his learners' learn how to add fractions, using a variety of activities, told us about his experience with subtraction of fractions. He assumed that, since the children had thoroughly understood how to add fractions, he need not spend much time on teaching subtraction. So he gave the learners' some examples of subtraction on the board, and then gave them the following exercise to do:

i) $\frac{2}{3}-\frac{1}{3}$, ii) $\frac{2}{3}-\frac{1}{5}$

In the following exercise we give you some more details and ask you to analyse this situation.

E4 A class was given problems (i) and (ii) given above to solve. Some children gave the answers $\frac{1}{0}, \frac{1}{2}$ respectively.

i) According to you, what could be the possible reasons for the gap in understanding the subtraction rule?

ii) According to these children what is the rule that has to be applied for subtraction of fractions?

iii) What could be the possible reason for the children acquiring this faulty rule?

iv) What are the various activities and exercises that children can do to get rid of this misconception.

Let us now consider a very important part of fraction learning. As we have mentioned about other topics, in this case too the **language used** at each step is very important. You might have observed that we use imprecise phrases like 'is the same as' for 'equivalent fractions', or 'dividing into', quite frequently. Children hear them and start repeating them, without quite understanding what they mean. These phrases appear like 'classroom jargon'. It is difficult to determine to what extent such phrases affect the process of learning. What is certain is that children pick up and use the phrases, and we assume that they absorb the ideas at the same time. We do not know what meaning they have for the child, nor what image they present. As some of you might have observed, the algorithms for adding or multiplying fractions, are rich in such ill-defined and unhelpful phrases. Most children recited them, almost as if they were part of a jingle, and show evidence of having been rehearsed many times. Let us see some examples of how the children confuse the different phrases like 'turn over' and 'cancel' that they come across while applying different algorithms. Here we shall concentrate on addition of fractions.

I once asked my neighbour's daughter Shanti, studying in Class 5, to work out $\frac{2}{3}+\frac{3}{4}$.

Shanti : I have got to make the denominator the same. So I can turn $\frac{3}{4}$ over. Then I get

$$\frac{2}{3}+\frac{3}{4}=\frac{2}{3}+\frac{4}{3}=\frac{6}{3}=2$$

So the answer is 2.

Here not that she has understood that, to add, she has to make the denominator the same. At the same time she has been told to turn over the fraction while dividing two fractions. So she wrongly applied the 'turn over' method to make the denominator the same. How would you correct her?

Another phrase which is commonly used in algorithms is 'cancelling. Here is an example where children wrongly apply this word.

Rita, when asked to work out the sum $\frac{2}{3}+\frac{3}{4}$, did it in the following way:

$\frac{\cancel{2}^{1}}{\cancel{3}_{1}}+\frac{\cancel{3}^{1}}{\cancel{4}_{2}}=\frac{2}{3}$

Hmm..... The terms at the bottom have got to be the same. You could cancel down first..... two goes into itself once, and two goes into four twice. Hm..... three goes inot three once, so that makes it $\frac{2}{3}$. (See her solution in the Margin.)

'Terms at the bottom have got to be the same', did you notice her telling herself this. This is another phrase commonly used in addition. Most of the children make the denominator the same without knowing why they are doing so. Here I quote an interesting instance that I read about.

A few children were asked to work out the sum $\frac{4}{3}+\frac{5}{4}$. One child did it in the following way:

$$\frac{4}{3}+\frac{5}{4}=\frac{16}{12}+\frac{15}{12}=\frac{31}{12}.$$

Teacher : Could you explain why you used 12?

Girl : To make the bottoms the same.

Teacher : And why do you have to make the bottoms the same?

Girl : Because you can't add if they are thirds and fourths that is you have to find the number that three and four will both go into. And then how many times three will go into 12, four, and then four times four sixteen..... And that's how $\frac{31}{12}$.

Teacher : Fine, instead of writing $\frac{2}{3}+\frac{3}{4}$, you wrote $\frac{16}{12}+\frac{15}{12}$. Somebody could think that was a different sum you are doing?

Girl : Yes

Teacher : What would you say to them?

Girl : You have to put them in twelfths if they are not the same.

Teacher : But someone might think you're doing a different sum?

Girl : Yes..... it's just the way we've been taught.

The point that emerges from the discussion above is that we **should always provoke our learners to think about why they are following a particular procedure.** For this we ourselves have to develop a habit of asking them questions so that it gives them a chance to think about what they are doing. We must also provide an environment in which they can feel that any question articulation or confusion they voice or reject is not to be laughed at or ignored.

E5 When given the problem "Subtract $1\frac{1}{2}$ from $3\frac{2}{5}$, a child does $1\frac{1}{2} - 3\frac{2}{5}$. Why do you think the child was making this error?
How would you help her to get over it?

Let's look at another situation.

Example 5: One day my neighbour came to me with her daughter. She looked worried. Her daughter was not able to convert mixed fractions into improper fractions. For instance she was not able to write $1\frac{1}{3}$ as $\frac{4}{3}$. She told me that she tried to explain it to her the way it has been done in the textbook. She showed me the method given in the textbook. It says that $1\frac{1}{3} = \frac{3 \times 1 + 1}{3} = \frac{4}{3}$.

After going through the text I interacted with the child.

I : Do you know how to find the sum of two fractions?

Girl : Yes.

I : O.K.! Would you find the following sums for me?

$\frac{1}{2} + \frac{3}{2}$, $\frac{1}{4} + \frac{1}{3}$, $\frac{3}{2} + \frac{2}{5}$.

She did so, correctly.

I : Now tell me, what is $\frac{2}{2} + \frac{1}{2}$?

Girl : It's $\frac{3}{2}$.

I : And what is $\frac{3}{3} + \frac{1}{3}$?

Girl : It's $\frac{4}{3}$.

I : But isn't $\frac{2}{2} + \frac{1}{2}$ the same as $1 + \frac{1}{2}$.

Girl : It's the same.

I : Can we write $1 + \frac{1}{2}$ as mixed fraction $1\frac{1}{2}$?

Girl : Yes we can do that.

I : So that means $1\frac{1}{2} = 1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$. Now can you give me an example of a mixed fraction?

Girl : $2\frac{1}{2}$.

I : Why is it called a mixed fraction?

Girl : Because it is the same as $2 + \frac{1}{2}$. It consists of 2 wholes and one part of a whole which is divided into two parts.

I : Very good.

Then I made her write a few more numbers like $1\frac{1}{2}$, $3\frac{1}{2}$, $3\frac{1}{3}$ in the form

$$1\frac{1}{2} = 1 + \frac{1}{2}$$

$$3\frac{1}{2} = 3 + \frac{1}{2}$$

$$4\frac{1}{3} = 4 + \frac{1}{3} \text{ etc.}$$

I : Now can you tell me what is $1 + 1 + \frac{1}{2}$.

Girl : Hmm..... it is one whole, then one more whole and half.

Yes! is's $2\frac{1}{2}$.

I : And how much is $1 + 1 + 1 + \frac{1}{2}$.

Girl : It's $3\frac{1}{2}$.

I : Good! Now let's see about converting mixed fractions into improper fractions. You just saw that $1\frac{1}{2} = 1 + \frac{1}{2}$. What is the improper fraction corresponding to $1\frac{1}{2}$?

Girl : It's $\frac{3}{2}$.

I : How did you get it?

Girl : Because $1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$.

I : O.K.! Now what is $2\frac{1}{2}$?

She looked a bit confused so I tried to help her. I wrote $2\frac{1}{2} = 1 + 1 + \frac{1}{2}$.

I : Isn't $1 + \frac{1}{2} = \frac{3}{2}$?

Girl : Yes, auntie.

I : So $2\frac{1}{2} = 1 + \frac{3}{2} = \frac{2}{2} + \frac{3}{2} = \frac{5}{2}$.

Girl : Oh! that's easy.

I : Now you write the improper fraction for $4\frac{1}{2}$.

Girl : $4\frac{1}{2} = 1 + 1 + 1 + 1 + \frac{1}{2}$

$= 1 + 1 + 1 + \frac{3}{2}$

$= 1 + 1 + \frac{5}{2}$

$= \frac{2}{2} + \frac{2}{2} + \frac{5}{2} + \frac{9}{2}$

I : Good. Now if I want to convert $1\frac{1}{3}$, then $1\frac{1}{3} = 1 + \frac{1}{3}$. What is the next step? (I gave her a hint) You know that $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4}$.

The girl took no time in understanding and immediately wrote $1\frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$.

Likewise, she wrote $2\frac{1}{3} = 1 + 1 + \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$. After this I asked her to do the same for $4\frac{3}{5}$. She wrote

$$4\frac{3}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = \frac{23}{5}$$

Slowly I led her to the quicker way of obtaining improper fractions. By looking at the number patterns, she discovered that

$$4\frac{3}{5} = \frac{5 + 5 + 5 + 5 + 3}{5} = \frac{4 \times 5 + 3}{5} = \frac{23}{5}.$$

Then she wrote the improper fractions corresponding to many more mixed fractions like $3\frac{1}{4}$, $7\frac{5}{8}$, etc. correctly. She was visibly happy and looked confident.

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After going through, this example, you may like to try the following exercise.

E6) Based on your experiences and Example 5 do you agree that "minor errors, which can be easily tackled by an understanding teacher, can block a child's understanding of the entire area."? Give reasons for your answer.

The example shows us how children often have minor points of misunderstanding, which the teachers can sort out, in a very short time, if they care to. Unfortunately many of us ignore such small points and carry on teaching other aspects of the concept, concerned. We do not realise that this often leads to a mental block in their understanding of the concept and the related concepts. The important thing is that having a dialogue with the child helps her get over the problem. This particular example may or may not work on some one. For each situation and context you would have to start from what the learner knows and move towards clarifying the concept that she is learning.

In the next section we shall take up some problems in multiplication and division of fractions.

14.3 DEVELOPING UNDERSTANDING IN MULTIPLICATION AND DIVISION

In this section we shall discuss ways in which we can help children to understand how to multiply and divide fractions. The need for such a section arises because of problems like the following that you may be familiar with.

Children often have a lot of problems while trying to simplify expressions like $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$. This could be because in their school they have only been given rules for simplifying such expressions. Those who can remember the rules can get a correct answer whereas some don't even know where to begin from. It is also seen at times that when children are asked why $\frac{1}{4}$ is the same as $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$, they simply reply 'by cancelling'. While trying to explain how they simplify such an expression, they are not able to decide which "whole" to use. Their attempts at explaining reveal their lack of background.

Children should be helped to realise that before they find two- thirds of something they need to have picture of what that 'something' is. This means that they should know that the "whole" is. How can we help the children in this regard? Let's see how my friend, a teacher in a school, introduces multiplication of fractions to her learners.

Example 6: Before multiplication the teacher wanted to make it sure that whether students can relate a "part" to a "whole". She checked it by giving exercises with different kinds of whole, the way we have discussed in Unit 12. Then she proceeded like this.

Teacher : Let us see what $\frac{2}{3}$ of $\frac{4}{5}$ is. $\frac{2}{3}$ of $\frac{4}{5}$ means $\frac{2}{3}$ of $\frac{4}{5}$ of 1 whole. So we shall first find $\frac{4}{5}$ of 1. Let's use paper shading to see this. [Teacher takes a big paper and pastes it on the blackboard using board pins or cello tape (See Fig.9), she draws a rectangle on this paper, this rectangle stands for 1 whole.] Let's first find $\frac{4}{5}$ of 1. [She asks one girl Shalini to come to the board and shade the portion $\frac{4}{5}$ of 1.]

Shalini : To find $\frac{4}{5}$ of 1, we have to divide the whole into 5 parts and then shade (by filling with lines) 4 parts out of 5. (She did as in Fig.9).

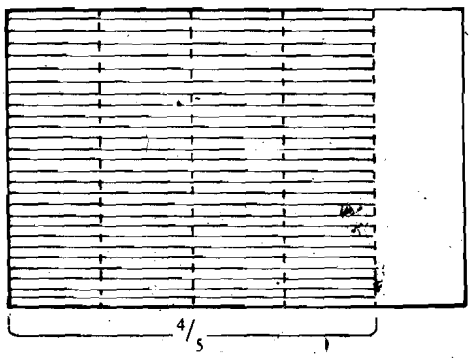

Fig.9: Pictorially showing $\frac{4}{5}$ of 1.

Teacher : Fine! Now what is $\frac{2}{3}$ of this portion? How do we find that? Remember,

we have to find $\frac{2}{3}$ of $\frac{4}{5}$, not $\frac{2}{3}$ of one. (She points at the shaded portion.) How do we do this? Ravi, can you try?

Ravi : We have to divide that into 3 parts and consider 2 parts out of it.

He tried to divide that vertically into 3 parts.

Teacher : Would it be easier if you do it horizontally?

Ravi : Hmm, yes! That's easier.

Then he did it (see Fig.10)

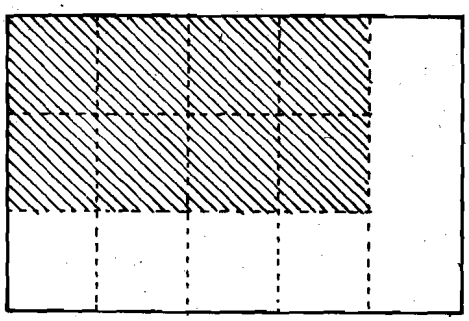

Fig.10: A pictorial representation of 2/3 of 4/5 of 1.

Teacher : Good! So the shaded portion is $\frac{2}{3}$ of $\frac{4}{5}$. Now we want to find what fraction this is of 1 (the original whole). To find this out easily we complete the partition of the whole like this (see Fig.11).

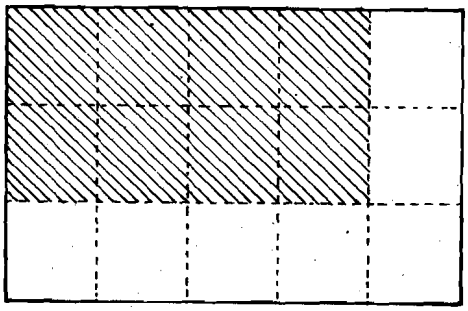

Fig.11: $\frac{2}{3}$ of $\frac{4}{5}$ of 1 is the same as $\frac{8}{15}$.

Now tell me how many parts have we divide up the whole into? Or, how many boxes are there in total?

Children : There are 15.

Teacher : How many boxes are there in the shaded portion.

Children : 8 boxes.

Teacher : So that means the shaded portion represents $\frac{8}{15}$ of the whole.

So, $\frac{2}{3}$ of $\frac{4}{5}$ is $\frac{8}{15}$.

The children agreed. The teacher then asked them to from 5 groups A,B,C,D and E. She gave each group a different problem to do. She spent quite a bit of time with each group, checking whether the children had understood how to find a fraction of a fraction using shading.

Then she tried to get them to evolve the rule using the pattern of the answers they

got. For this she listed the problems given to each group on the blackboard with the answer they got, alongside, as below.

A : $\frac{5}{7}$ of $\frac{2}{3}$ is $\frac{10}{21}$

B : $\frac{2}{5}$ of $\frac{3}{4}$ is $\frac{6}{20}$

C : $\frac{3}{8}$ of $\frac{2}{3}$ is $\frac{6}{24}$

D : $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{6}{12}$

E : $\frac{4}{7}$ of $\frac{3}{5}$ is $\frac{12}{35}$

Teacher : Look at the questions and their answers. Is there anything you notice about 7 and 3 in the first question and the 21 you got in your answer?

One Student : Yes, Ma'm, 7×3 is 21.

Teacher : And what about 5, 4 and 20 in the next question and the rest of the questions of C,D and E? Do you notice some pattern?

Child : Yes 5×4 is 20, 8×3 is 24, 3×4 is 12 and 7×5 is 35.

Teacher : Do all of you agree with this?

Children : Yes, Ma'm.

Another Student : Isn't the numerator 10 in the first question because it is 5×2?

Teacher : Excellent. Now can you say, without shading what $\frac{3}{4}$ of $\frac{5}{6}$ is?

Same Student : It is $\frac{3 \times 5}{4 \times 6} = \frac{15}{24}$.

Teacher : Very good. So, now know how to multiply fractions. You multiply the numerators to get the numerator and the denominators to get the denominator.

Once they got used to the procedure for multiplication, she made it a point to take them back to addition of fractions, and help children to realise that, for example

$$\frac{2}{3} + \frac{4}{5} \neq \frac{2+4}{3+5}.$$

[It is **important** to bring out such differences which the children may not realise otherwise.]

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So you saw how students arrived at the rule for multiplication. Note that before such 'rule forming' or 'pattern generalisation' can take place, children need to have a lot of activity-based experience in finding $\frac{a}{b}$ of $\frac{c}{d}$. This gives them confidence because they can then explain why the rule is true.

Why don't you try some exercises now?

E7) Suggest some activity to help children find $\frac{2}{3} \times \frac{3}{4}$.

Next we shall discuss some problems that arise in cancelling the common factors in the multiplication of fractions which we have discussed earlier. While solving problems some children wrote $\frac{4}{5} \times \frac{5}{6} \times \frac{6}{4} \times \frac{7}{6} = 7$,

Note that here they have cancelled both the sixes in the denominator. Another error which is commonly seen is that after cancelling the common factors they write 0 instead of writing 1. Such errors can be avoided if they clearly understand that cancelling means dividing by a common factor. Also, they need to realise that corresponding to one 5 in the numerator they can cancel only one 5 in the denominator, with one 6 in the numerator only one 6 in the denominator and so on. In the next example, let's see how teacher explains the cancelling process to her students.

Example 7: The teacher interacts with the students in the following way:

Teacher : What is $\frac{3}{5} \times \frac{5}{8}$.

Students : $\frac{3}{5} \times \frac{5}{8} = \frac{3 \times 5}{5 \times 8} = \frac{15}{40}$

Teacher : Can you simplify it?

Students all together : It is $\frac{3}{8}$.

Teacher : Good, Now let me ask you another question. Is 3×5 the same as 5×3.

Students : Yes

Teacher : So how else can you write $\frac{3 \times 5}{5 \times 8}$?

One Student : $\frac{3 \times 5}{8 \times 5}$

Another Student : $\frac{5 \times 3}{5 \times 8}$

Another Student : $\frac{5 \times 3}{8 \times 5}$

Teacher : O.K.! Let's take $\frac{5 \times 3}{5 \times 8}$. Can you write this as the product of two fractions?

Student : Yes. $\frac{5 \times 3}{5 \times 8} = \frac{5}{5} \times \frac{3}{8}$.

Teacher : What is $\frac{5}{5}$?

Students : $\frac{5}{5} = 1$.

Teacher : Then we can write the product as

$$\frac{5 \times 3}{8 \times 5} = 1 \times \frac{3}{8} = \frac{3}{8}$$

Another way of doing it is to divide the numerator and denominator by 5. Now, if we divide the numerator by 5, we get 1×3, and the denominator becomes 1×8. That is

$$\frac{5 \times 3}{5 \times 8} = \frac{1 \times 3}{1 \times 8} = \frac{3}{8}$$

Note that, after cancelling 5 from the numerator and denominator don't forget to write 1 in their places as I have written.

In the same way we can write

$$\frac{3 \times 5}{5 \times 8} = \frac{3 \times 1}{1 \times 8} = \frac{3}{8}.$$

This process of dividing the numerator and denominator by 5 is called cancelling 5 from numerator and denominator.

Next the teacher explained how cancelling is done for solving problems like $\frac{4 \times 7}{5 \times 6}$ in the same interactive way as before.

$$\frac{\quad}{\quad} \times \frac{\quad}{\quad}$$

From this example we gather that we need to explain the basics involved in the cancellation process in detail rather than just giving the process. Unfortunately, what teachers usually do is just discuss one or two problems in the class and give a lot of exercises as homework. This does not really serve the purpose because, in most of the cases, the children haven't understood the process. So they ask for help from their parents. And many parents are incapable of correcting children's mistakes. Therefore, before giving exercises to the students we need to ensure that they have understood the concept. The same is true if a parent, or any adult, is teaching a child.

Another problem which normally occurs in multiplication is the following:

While multiplying two mixed fractions say $2\frac{1}{2} \times 3\frac{2}{3}$, children write

$$2\frac{1}{2} \times 3\frac{2}{3} = 2.$$

This happens because of their wrong notion that $2\frac{1}{2} = 2 \times \frac{1}{2}$ and $3\frac{2}{3} = 3 \times \frac{2}{3}$. To avoid this, we need to convince them that $2\frac{1}{2} \neq 2 \times \frac{1}{2}$. One way of doing this is to give them a variety of concrete experiences to show them that while multiplying two mixed fractions they have to first convert them into improper fractions and then multiply them.

How will you do that? We put it as an exercise for you.

E8) Suppose a child has got the wrong notion that $2\frac{1}{2} = 2 \times \frac{1}{2}$. How will you convince her that she is wrong and that $2\frac{1}{2} = 2 + \frac{1}{2}$?

So far we have been discussing various problems related to the multiplication of fraction. Let us now consider some problems related to division.

In this regard let me share the experience I had with my 10-year- old neighbour Ajitha, and her friend. We were discussing problems on operations with fractions. During our discussion I asked them what $\frac{1}{3} \div \frac{1}{4}$ is. Ajitha immediately said, that she knew how to find this: $\frac{1}{3} \div \frac{1}{4} = \frac{3}{4}$.

Hearing this her friend Renu commented that it isn't $\frac{3}{4}$, it's $\frac{4}{3}$. I asked them how they got these answers:

Ajitha : $\frac{1}{3} \div \frac{1}{4} = 3 \times \frac{1}{4}$.

Renu : But this is wrong, we have to invert the second number and then multiply.

Ajitha : Oh Yes! I made a mistake. It is $\frac{1}{3} \times 4 = \frac{4}{3}$.

From their discussion I gathered that they hadn't understood the basics involved in the divisions process. So I started explaining to them.

I : Suppose you have 12 toffees and you want to share these equally among your 4 friends, then how many toffees will each of you get?

Girls : 3 toffees.

I : Can I write it as $12 \div 4 = 3$?

Girls : Yes.

I : So $12 \div 4$ tells us that how many groups of 4 we can take out of 12.

Then I brought 4 glasses. Out of these 4, 2 glasses were empty and two were full of lime juice. I poured the juice into a pot.

I : Now I want you to distribute this juice equally among 4 of us—You two, me, and the maid.

They started pouring the juice into four glasses equally. They found that each glass was $\frac{1}{2}$ filled.

I : How many $\frac{1}{2}$ glasses could you take out from 2 glasses of juice.

Girls : 4.

I : That means $2 \div \frac{1}{2} = 4$. Do you agree with this?

Girls : Yes.

Then I brought a ribbon and measured 2 metres from that. Then I measured $\frac{1}{2}$ metre and cut it. I asked them how many of these $\frac{1}{2}$ metre pieces could be cut from the 2 metre piece.

Girls : 4 pieces (They showed me how they got the answer).

I : Very good. So we get that $2 \div \frac{1}{2} = 4$. Now, note that

$2 \div \frac{1}{2} = 4$ and also $2 \times 2 = 4$

Isn't it.

Girls : Yes.

I : Now, let's see what $6 \div \frac{1}{3}$ is.

I took a long ribbon and showed them that $6 \div \frac{1}{3} = 18$. At the same time I drew their attention to the similarity between

$6 \div \frac{1}{3} = 18$ and $6 \times 3 = 18$.

Then continued my discussion like this.

I : Now can you say what $9 \div \frac{1}{3}$ is without using the ribbon.

Girls : Is it 27, madam?

I : How did you get it.

Girls : $9 \times 3 = 27$.

I : Very good.

I clearly explained to them the rule that "Division of a fraction by another fraction is its multiplication by the reciprocal of the second fraction. Then I made them do a variety of problems like the following to them practising the rule:

$$12 \div \frac{4}{3} = 12 \times \frac{3}{4} = 9$$

$$\frac{1}{3} \div \frac{1}{4} = \frac{4}{3}$$

Later I showed them some of the cases like $\frac{1}{3} \div \frac{1}{4} = \frac{4}{3}$ using a ribbon. This helped them to get a logical explanation of the rule.

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Now here is an exercise for you, related to what we have been discussing.

E9) How would you use paper folding activities to explain to a child that $1 \div \frac{1}{2} = 2$ and $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$.

With this we come to the end of this unit. We hope that this discussion has helped you to understand why students face lots of problems while performing arithmetical operations on fractions. Through this unit we have emphasised the need for the child to understand the concept before she is given the algorithms. We hope that the various activities discussed in this unit will help you to help the children around you to deal with fraction. We now summarise what we have covered in this unit.

14.4 SUMMARY

In this unit we have

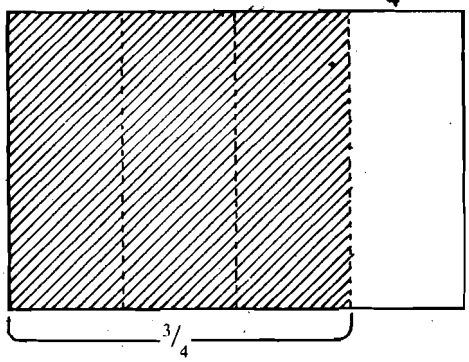
1) focussed on some common errors that children make while applying the four fundamental operations with fractions.

2) discussed some ways of helping children overcome certain problems that they may face while learning how to apply the operations.

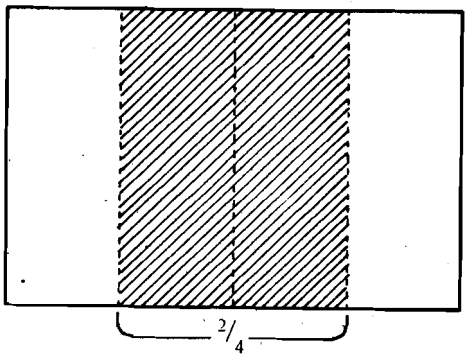
14.5 COMMENTS ON EXERCISES

E1) Paper folding is an effective method for teaching addition of like fractions. Whereas, if the fractions to be added are unlike, it is difficult to explain with paper folding. For example, suppose we want to find $\frac{1}{8} + \frac{1}{3}$, then the total no. of parts into which the paper is divided should be 8 for showing $\frac{1}{8}$ and 3 for showing $\frac{1}{3}$. This becomes difficult to show on the same piece of paper. If we use two different pieces of paper for showing $\frac{1}{8} + \frac{1}{3}$, there is a chance that a child may get confused with the "whole" under consideration.

Paper folding can be used to teach $\frac{3}{4} - \frac{1}{4}$. For this, divide the whole sheet into 4 parts. Shade 3 parts with the pencil, which shows $\frac{3}{4}$ (See Fig. 12(a)).



(a)



(b)

Fig.12: Showing $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$.

Now we remove the shade from one part among the three parts shaded. This shows $\frac{3}{4}-\frac{1}{4}$ (see Fig. 12(b)).

E2) A class of 30 students has been divided into 5 group. Each group has some teaching aids, which are paper strips, coloured pencils, bread pieces etc. We shall explain the group activity using bread pieces. Suppose that Group B is doing this activity. Give a loaf of bread to Group B and ask two of the children to give 1/4 each of the loaf. Talk to them and get them to tell you that how they know that the portion is 1/4. Each time you should explain to them that to get 1/4 of the bread, they have to divide the bread into 4 equal parts and then take one part.

Now ask them how much you have got in total. Once they are convinced that it is 1/2, you ask them to find $\frac{3}{4}+\frac{1}{4}$ using the same activity. Then give a variety of problems, using various objects.

In this way you could interact with other groups. After interacting with all the groups, you could give a problem which each group can try. Then call a representative from each group and ask them to explain what they have done.

E3) You might agree with us that the activity using fractions charts is very helpful in understanding addition with unlike fractions. Especially the idea of finding a common whole can be explained clearly by a fraction chart. In the beginning children might find this activity a title difficult to practise, and some may get a little board. Once they learn to operate with the charts, they become more and more interested in this activity. Also, this activity has got the advantage that it can be used for explaining many other concepts related to fractions. So, why don't you try the activity and check its effectiveness yourself.

E4) In both the situations, the procedure they have applied is indeed subtraction the numerators and denominators i.e. $\frac{a}{b}-\frac{c}{d}=\frac{a-c}{a-d}$. You can note that in the first case, the children did not notice that division by 0 is undefined and in the second they have wrongly calculated 3–5. This shows that in both these cases there are several errors involved. Firstly, they have applied the rule wrongly. Secondly, while applying their version of the rule, they have committed an error. These problems may arise due to the fact that the teacher did not spend enough time to teach subtraction of fractions. He assumed that since they have understood addition, subtraction follows on the same lines. This fact may be true for brighter students. But for an average child we may have to spend the same amount of time, probably more, to make them understand the complexities involved in the subtraction of fraction.

The activities that can be tried are paper folding activities or those using fraction charts.

E5) You must have noticed that whenever someone is trying to teach a child how to apply operations on whole numbers or fractional numbers, they concentrate on trying to ensure that the child learns the algorithm. In the process, unfortunately, they tend to ignore the problems that can arise due to the lack of understanding of the language involved. The language may be English (or Hindi, or.....) or may be it is the language of Mathematics. For example, when you tell a child to subtract, say 2/3 from 3/2, you assume that the child understands the meaning of 'subtract' as well as the meaning of the use of the word 'from'. So a child needs a lot of exercises and practice before she can absorb the meaning of various words and phrases.

E6) We have discussed some hints for this, a little further after E6, in the main text.

E7) Paper folding activity can be used to show the meaning of $\frac{2}{3}\times\frac{3}{4}$ and to explain the method of finding $\frac{2}{3}\times\frac{3}{4}$.

Take two strips of paper. Divide one strip into 4 equal parts and cut out 3 parts from it (See Fig.13 (a), (b)).

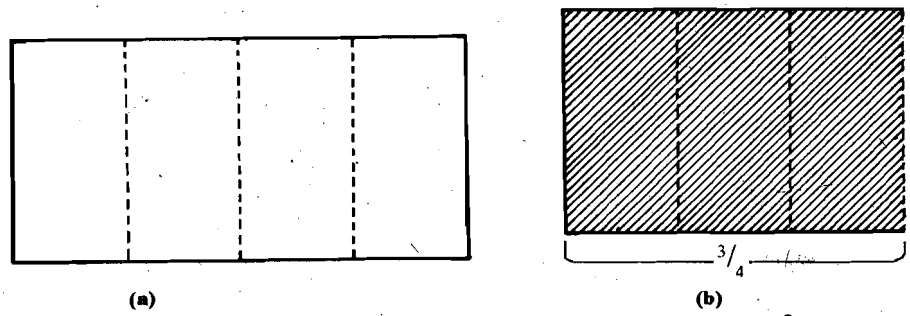


Fig.13: (a) shows partitioning '1' into 4 equal parts, (b) shows the cut out portion $\frac{3}{4}$ of 1.

The cut out portion of the strip represents 3/4. Now, to find $\frac{2}{3} \times \frac{3}{4}$, which is $\frac{2}{3}$ of $\frac{3}{4}$, we find the two-third portion of the cut out portion. For that we fold the cut out portion into 3 equal parts and again cut 2 parts from that (see Fig. 14a).

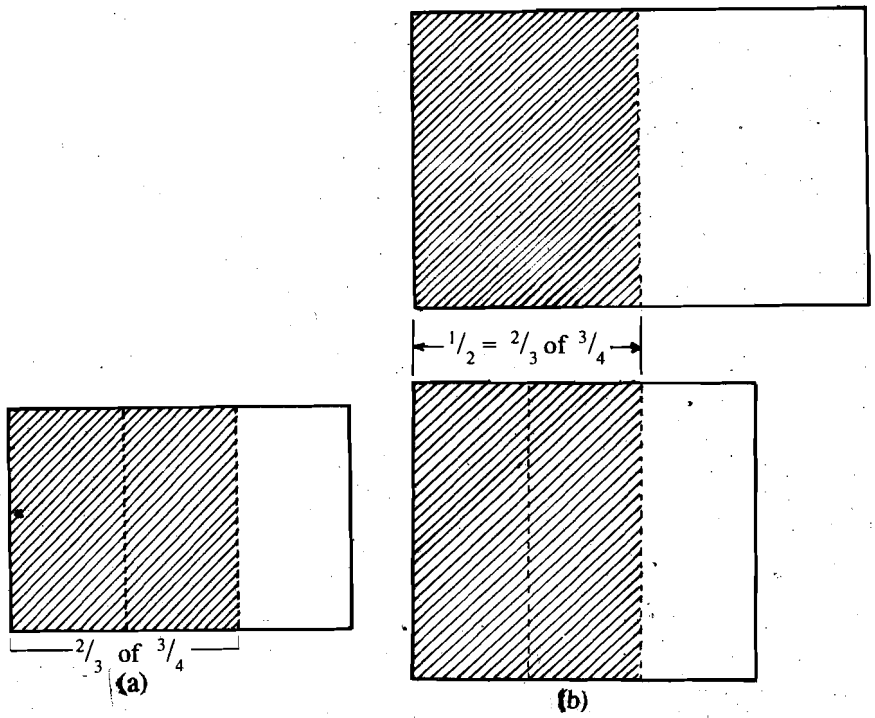


Fig.14: (a) Pictorially representing $\frac{2}{3}$ of $\frac{3}{4}$ of 1, (b) showing $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

Now, to see how much is the portion (the portion shown in Fig.14 (a)) we compare it with the strip which is kept unfolded. For that we place both the strips, one below the other (see Fig.14 (b)) and observe that the cut out portion is $\frac{1}{2}$ of the whole strip. This shows that $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

You could repeat the same procedure for finding $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$. This is only one type of activity. Of course, you could think of many more.

E8) Suppose a child commits the error $2\frac{1}{2} = 2 \times \frac{1}{2}$. She may know the meaning of 'two and a half', but is confused about the notation. If the problem is only of notation, she requires a lot of practice in using it. If the problem is that she has not understood the meaning of 'two and a half', then you need to give several activities to help her understand. You can give 2 biscuits and $\frac{1}{2}$ a biscuits, or 2 leaves and half of a leaf etc. and ask them how much they have

got. Similarly to get across the meaning of '$2 \times \frac{1}{2}$' (if necessary), you can give a variety of activities. Give them two half biscuits or two halves of a leaf and ask them how much they have got. At the end of this you give a few pages and ask her to give you $2\frac{1}{2}$ pages first and then $2 \times \frac{1}{2}$ pages. This will allow you to check whether she has understood the difference between $2\frac{1}{2}$ and $2 \times \frac{1}{2}$.

E9) Take two strips of paper. Keep one without folding. Fold the second paper into 3 equal parts and shade each part differently with a pencil (You can use three different coloured shades also). Now ask the child how much each shaded part represents. She will say $\frac{1}{3}$. Next place the two strips one below the other as shown in Fig.15.

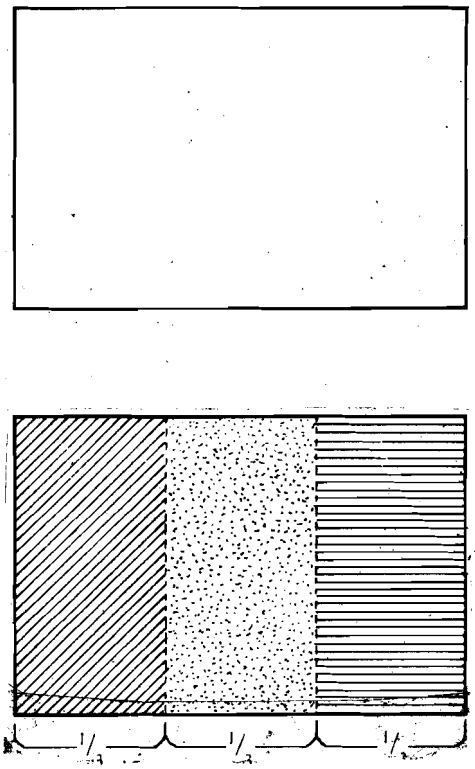


Fig.15: Pictorially showing $1 \div \frac{1}{3} = 3$.

Then she can observe that there are three $\frac{1}{3}$'s in a whole; which eventually is the meaning of $1 \div \frac{1}{3}$.

Now to show 3/4÷ 2/4, we can first explain to them $\frac{3}{4} \div \frac{1}{4}$. For that take 2 strips, divide both the strips into 4 equal parts. Shade 3 parts on one strip, which shows $\frac{3}{4}$, and shade one part on the other strip which shows $\frac{1}{4}$. Then place both the strips one below the other (see Fig.16). Mark the portion showing $\frac{3}{4}$ by A and that showing $\frac{1}{4}$ by B.

Now ask the children how many times the portion B fits into A. You can cut the portion B and physically show that it fits into A, 3 times. This means that the number of times $\frac{1}{4}$ goes into $\frac{3}{4}$ is 3 which is the same as the number of

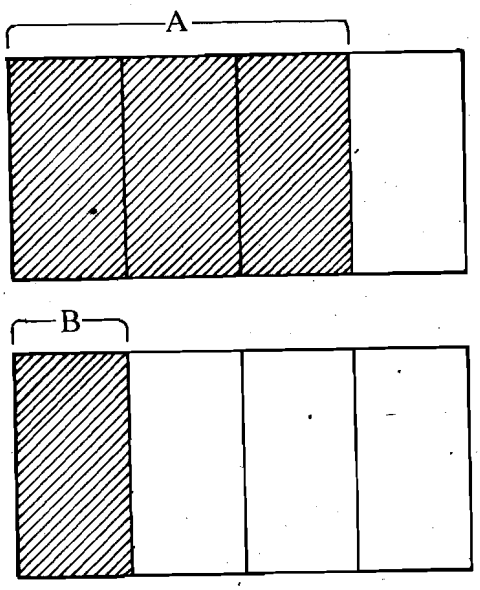


Fig.16

times one goes into 3. Hence the question, "how many $\frac{1}{4}$'s are there in the number $\frac{3}{4}$?" is the same as the question "how many 1's are there in 3?".

Therefore we get $\frac{3}{4} \div \frac{1}{4}$.

$$= 3 \div 1 = 3$$

Similarly you can explain that $\frac{3}{4} \div \frac{2}{4} = 1\frac{1}{2} = \frac{3}{2} = 3 \div 2$.

UNIT 15 DECIMALS

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15.1 INTRODUCTION

In the last unit we discussed some problems children face while operating with fractions. In this unit we concentrate on problems children face when fractions are represented by decimal numbers, i.e., numbers that are written with a decimal point.

The concept of 'decimals' is an important concept which children normally find difficult to grasp. As a result they commit errors while dealing with decimal numbers. For example, when asked which of the numbers .05, .5, .5808, .056 is the smallest, a 15-year old child tried to remember the rules, she has learnt in Primary school. A little later she said .5808. When asked why she chose this number, she said that the teacher taught her that the more the digits after the decimal point, the smaller the numbers.

To cite another example, when 12 to 15-year-old children are given the problem $40 \div 0.8$, you are likely to get answers like 5 or 0.5. In this unit we shall look at why children have such problems with decimals. We shall also try to evolve some appropriate teaching strategies for solving them.

Regarding terminology decimal numbers are also called decimals or decimal fractions. We shall stick to the term decimal numbers.

Objectives

After going through this unit, you should be able to

- use various materials that children usually use for introducing the concept of decimal numbers;
- identify ways in which you can help children understand the place value representation of decimal numbers;
- suggest teaching methods which help children evolve the rules themselves for applying the four operations to decimal numbers;
- create various activities to help children improve their computational skills with decimal numbers.

We first discuss some problems that arise because the children have not understood the decimal notation.

15.2 WHAT IS A DECIMAL NUMBER?

While learning/teaching decimals how many of us ask questions like "Why is 1.5 the

same as 1.50 or 1.500 and not the same as 1.05?" If you ask a child who has "learnt decimals" this question, you would most probably find them looking confused and uncertain.

In this section we shall discuss some ways of trying to improve their understanding of decimal representation so that such confusion does not arise. Let's look at a situation regarding this.

Example 1: I was talking to a Class 4 teacher of a school nearby about how she teaches the children decimal representation. She told me that she first familiarises them with the idea of decimal notation by giving them the rule for division by 10. For example, she writes this as

$$\frac{7}{10} = .7$$

$$\frac{27}{10} = 2.7 \text{ etc.}$$

$$\frac{125}{10} = 12.5$$

So the rule is that 'dividing by 10' is the same as putting the decimal point before the last digit. She, then, extended this rule to fractions with denominators 100 and 1000, for example, writing $\frac{27}{100} = .27, \frac{205}{100} = 2.05, \frac{127}{100} = .127$, etc. She also brought out the fact that .7 = .70.

Some time later, she would give the class problems to do, to evaluate their understanding. For instance, one of the batches was asked to write 3/100 in decimal form. Some of the children were confused by this because they did not know where to get the second digit from. She explained to them that you start with the number and move leftwards, wherever there are no digits put a zero. She explained to them what is the difference between .03 and .3. Then she gave them several exercises like converting $\frac{19}{1000}$ for practice.

Before we go further explaining what the teacher told us, we would like you to try an exercise.

E1) Suppose a child reads 0.06 as zero point six. How will you convince her that this is wrong, and that it should be read as zero point zero six?

While trying these exercises you must have felt that it is necessary to know the place value representation of decimals to get a clear understanding of decimal notation. Let us see how the teacher introduces her children to the place value representation of decimals. Note that she did this **only after** giving them enough practice with the rules for conversion from decimal representation to fractions and vice versa.

First she started with some whole numbers, say 324, 271 and 450. She asked the students to write these numbers as multiples of hundred, ten and one. They did this easily, writing

$$324 = 3 \times 100 + 2 \times 10 + 4 \times 1$$

$$271 = 2 \times 100 + 7 \times 10 + 1 \times 1$$

$$450 = 4 \times 100 + 5 \times 10 + 0 \times 1.$$

Next, she pointed out the relationship between the adjacent places to them, saying, "Look at the "hundreds" place and the "tens" place. How will you get 10 from 100? By dividing 100 by 10. Look at the "tens" place and the "ones" place. How will you

get 1 from 10? - By dividing 10 by 10". Pointing to each place in turn, she said, "So, as we move rightward, we divide by 10. In the same way we can continue the places after the "ones", by dividing each time by 10. So the place to the right of the "ones" will be "one-tenths", and then "one-hundredths". And to show, where the "ones" stop, we put a dot after the "ones". For example,

2.5 means $2 \times 1 + 5 \times \frac{1}{10}$

$.7$ means $7 \times \frac{1}{10}$

11.47 means $1 \times 10 + 1 \times 1 + 4 \times \frac{1}{10} + 7 \times \frac{1}{100}$.

So 2.5 is $2 \times 1 + 5 \times \frac{1}{10} = 2 + \frac{5}{10} = 2 + .5$. And in the same way, 16.7 is 16 + .7. So the decimal point separates the whole number portion from the fractional portion.

Then she told me that after explaining these things, the next thing she usually did with the children was to make them analyse a number in the place value format, like for example,

$$51.6 = 51\frac{6}{10} = 5 \times 10 + 1 \times 1 + 6 \times \frac{1}{10}.$$

She would also follow the reverse procedure of converting a decimal number given its place value break up, as for example,

$$2 \times 100 + 0 \times 10 + 7 \times 1 + 8 \times \frac{1}{10} + 3 \times \frac{1}{100} = 207.83$$

After introducing the place value representation she gave them many problems to solve, including word problems like "A table is 92.3 cms. long. How many metres long is this?" Here I asked her why she does not want to give such problems earlier. For that she replied that whenever she gave such problems earlier, children tend to commit errors. For example, when asked how to write 8 paise in terms of rupees, they write .8, not .08 . So it is better to give such problem after having learnt the place value representation.

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So this is how she introduced decimal numbers to her learners. Now, we ask you to write your comment regarding the situation above in the following exercises.

E2) What are the positive and negative points of the teaching strategy in Example 1.

E3) From your experience, would explaining the logic behind the writing of decimal fractions (as powers of $\frac{1}{10}$) to a Class 4 child help her or confuse her? Justify your answer.

Let's look at another situation where a teacher is introducing decimals to Class 4 students.

Example 2: The teacher brought 5 unsharpened pencils and 5 rubbers. The length of each pencil was approximately 10 times that of a rubber (see Fig.1). She also brought strings of different lengths.

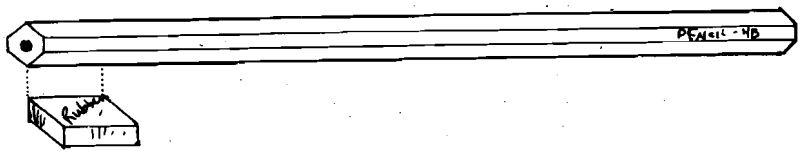


Fig.1

She divided the students into 5 groups. To each group she gave a pencil, a rubber and a piece of string. Then she asked them to measure the length of the string using the pencil and rubber. She helped to start them off. Let's see how she interacts with Group 1.

Table 1

Pencil	Rubber
3	6
2	5
1	3
4	7
5	1

Teacher : Use the pencil to find the length of this string; How many times the pencil is this string?

(They showed her and said 3. Some portion of the string was still left unmeasured.)

Teacher : What about the remaining portion?

Group : It is less than a pencil.

Teacher : Can you measure that portion with the rubber?

Group : It is 6 times the rubber.(They showed her.)

Teacher : So the string is 3 times the pencil and 6 times the rubber. Let me write this in a table (see Line 1 of Table 1).

Teacher : Now look at this table. The length of the string measured by Group 1 is 3 times the pencil and 6 times the rubber. Now, tell me how many times the rubber is the pencil. You measure it and see.

One Student : Ten times.

Teacher : How did you get it?

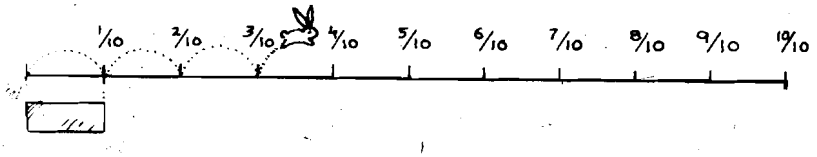
(She measured with the rubber and showed her.)

Teacher : What portion of the pencil is the rubber? (No answer). If ten rubbers are equal to one pencil, one rubber is equal to how much of the pencil?

Another Student : $\frac{1}{10}$?

Teacher : Yes it is $\frac{1}{10}$.

Then the teacher draw a line which is of the same length as the pencil and another small rectangle with the same length as of the rubber, as in Fig.2.



She marked $\frac{1}{10}, \frac{2}{10}$ ..... $\frac{1}{10}$ by moving the rubber along the line.

Teacher : So, the rubber is $\frac{1}{10}$ of the pencil. Then 6 times the rubber means 6 times $\frac{1}{10}$ which is $6 \times \frac{1}{10}$, i.e., $\frac{6}{10}$ of the pencil. Therefore, the string is 3 times the length of one pencil and 6 times length of $\frac{1}{10}$ of the pencil.

Table 2

Pencil (ones)	Rubber (tenths)
3	6

Then she wrote "ones" below the pencil and "tenths" (or $\frac{1}{10}$ ths) below the rubber. She drew Table 2 side by side Table 1 on the black board so that they can see both the tables simultaneously and wrote 3 in the ones column and 6 in the tenths colum

Teacher : So what is the length of the string in terms of the pencil as a whole.

One student : $3\frac{6}{10}$.

Teacher : Very good And what are the lengths of the other strings?

Same student : $2\frac{5}{10}, 1\frac{3}{10}, 4\frac{7}{10}$, and $5\frac{1}{10}$.

Teacher : For convenience, instead of writing $\frac{1}{10}$ each time we put a dot. For example, $3\frac{1}{10}$ is 3.6. This show 3 ones and 6 tenths.

So, $3.6 = 3 \times 1 + 6 \times \frac{1}{10} = 3\frac{6}{10}$.

So we have written $3\frac{6}{10}$ with a dot as 3.6. Now can each group tell me the length of the string that you have measured with pencil and rubber in this way with a dot?

Table 3

Pencil (ones)	Rubber (tenths)
3	6
2	5
1	3
4	7
5	1

Groups : 2.5, 1.3, 4.7, 5.1.

Teacher : Wonderful! Now let me write these numbers in the table (She draw Table 3 side by side to Table 2). The dot is called the decimal point. This notation for the numbers, using the dots is called the decimal notation.

Teacher : Now, look at Table 3. Suppose I write 1 in the ones column and 4 in the tenths column, what is that number in decimal notation?

One student : It is one point four.

Teacher : How will we write this as a fraction?

Same student : It is $1\frac{4}{10}$.

Teacher : How will you expand this in terms of ones and tenths.

Students : $1 \times 1 + 4 \times \frac{1}{10}$.

Teacher : Suppose I write 0 in the ones column and 5 in the tenths column, what is this number in decimal notation?

Another student : (after some thought): Is it zero point 5?

Teacher : Yes. What is the fraction corresponding to this number ?

Student : $0\frac{5}{10}$.

Teacher : Means what?

Same student : It is $\frac{5}{10}$.

Teacher : Why?

Same student : $0.5 = 0 \times 1 + 5 \times \frac{1}{10}$.

In this way the teacher interacted with the students to evaluate whether they had understood the decimal representation of a fractional number.

——— × ———

This is how the teacher introduced the idea of a decimal number to the children. Then she explained to them how to write decimal numbers as multiples of hundred, ten, one, one-tenth and one-hundredth.

A word of caution! Children might confuse the word 'tens' with 'tenths', and 'hundreds' with 'hundredths'. A child when asked 'how many tenths are there in 35.21 may write 2 tens. This doesn't necessarily mean that she has not understood the place value representation. So we have to be very careful in pronouncing these words, as well as in writing them. We also need to be careful in dealing with children's answers that involve these words.

The situation above indicates how we can use activities to conceptualise decimal numbers. **Of course, one activity is not enough. We should try a variety of activities.** In the following exercises we ask you to think of more activities for this purpose.

E4) Give at least two different exercises/activities that can be given to a group of 20 children in a class to help them improve their understanding of the decimal notation.

E5) In Examples 1 and 2, we have discussed two different methods of introducing decimals. According to you, which method is more effective for a child to have a good basic understanding of decimals?

E6) If a child is able to convert some fractions into decimal numbers and vice versa, it shows that the child has understood the decimal notation. True or false? Give reasons.

By now you must have understood that it takes time for a child to understand the concept of a decimal point and a decimal notation of a number. This conceptual understanding is very necessary for a child to perform different operations with decimal numbers. In the next section we shall discuss some of the problem related to this aspect.

15.3 OPERATIONS WITH DECIMAL NUMBERS

Have you ever tried to teach a child how to add or multiply decimal numbers. While doing so, were you only making sure of the child's speed and accuracy in applying the different algorithms? Unfortunately, most of us do this, instead of seeing whether they have understood the underlying concepts involved in these operations. But, as we have discussed in Block 1, we must concentrate on developing such an understanding. We also need to develop the habit of asking children questions that enable them and us to check their level of understanding. The kind of exercises we give them should assess their basic understanding. When they apply an algorithm to do an operation, we should ask them how they know that their answer is correct.

In this section we shall consider ways of helping children understand and apply the algorithms for the four basic operations on decimals.

We shall first discuss some problems related to addition and subtraction.

15.3.1 Addition/Subtraction

In a discussion with a group of primary teachers, we found that one of the problems regarding addition of decimal numbers which children face is in adding two or more decimal numbers which have different number of digits in their decimal part e.g., 0.123 + 1.1 + 1.101. One of these teachers gave us a detailed picture of how she teaches addition of decimal numbers to students of Class 5. Let's look at her strategy.

Example 3: The teacher had already taught the class how to add decimals like, say 2.7 and 1.8, 18.75 and 20.65. The next step was to teach them how to add decimals like 18.7 + 20.65. She did it in the following way.

She wrote 18.7 + 20.65 on the board. Pointing to the numbers she told children that "the first number has one digit after the decimal point and the second has two digits. So we re-write 18.7 = 18.70. Now let us add them up by writing them one below the other. She wrote the following on the board.

$$\begin{array}{r} 18.70 \\ +\ 20.65 \\ \hline 39.35 \end{array}$$

and then repeated that the sum of 18.7 and 20.65 was 39.35. She did one more problem like this, and then asked the students to solve

$$0{\cdot}05 + 0.101 + 0.3.$$

She found that many of them wrote

$$\begin{array}{r} 0.005 \\ 0.101 \\ +\ 0.300 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 0.05 \\ 0.101 \\ +\ 0.003 \\ \hline \end{array}$$

In each case she had to correct their mistake repeatedly.

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Here are some exercises for you.

E7) Why do you think the children were making this error? How would you help them to get over it?

E8) Ask some children around you to think of real-life situations in which they would need to add decimal numbers. Note down their responses.

If you look at the mistakes that the students have committed in Example 3, you will find that they have not understood why 18.7 is the same as 18.70 or 18.700, but not the same as 18.07. To be more precise, why are zeros at the end of the numbers after the decimal point added or removed whenever needed? So we have to explain these things clearly while teaching the rule for addition. Probably, if the teacher had done it the way, we have mentioned in the comments to E6, the child would have got some insight into what is behind the rule. Some people have found that if the students are introduced to decimals using the activities discussed in the previous section, they gradually understand the place value of each digit after the decimal point, which in turn will help them perform the operations in a better way.

Here we give an exercise in which we have given an activity for you to try.

E9) How would you use the square divided into 100 parts (see comments on E4) to help a learner develop the ability to add? Does this method work better than the one given in the comments on E7? If so, why? If not, why not?

Let's now discuss some problems related to subtraction of decimals. It has been found by teachers that more errors occur when the items were written in a line and especially in which the decimal numbers had varying numbers of digits after the decimal point, say 3.4 – 1.53. The types of errors were of the following kinds:

a)
$$\begin{array}{r} 1.53 \\ -\ 3.4 \\ \hline 11.9 \end{array}$$

b)
$$\begin{array}{r} 3.4 \\ -\ 1.53 \\ \hline .81 \end{array}$$

c)
$$\begin{array}{r} 3.4 \\ -\ 1.53 \\ \hline 1.93 \end{array}$$

In the following exercise we ask you to look closely at these errors.

E10) What is the probable logic by which the answers above have been arrived at? What kind of practice and exposure do children need to correct their framework of understanding?

E11) Does the activity of 'dividing the square into 100 parts' help students perform the subtraction of decimals in a better way? Give reasons for your answer.

As we mentioned in Section 3.6 of Unit 3, Block 1, children's errors are a natural and inevitable part of their process of learning. The errors are reflections of how they think and learn. On analysing the various errors related to addition or subtraction of decimal numbers, you might agree that these problems occur probably more due to the lack of understanding of the decimal notation on either side of the decimal than to any problem concerned specifically with subtraction. According to us a good remedy for this is to develop a sound understanding of the place value representation of decimals.

Let us now discuss some problems related to multiplication and division of decimal numbers that children commonly face.

15.3.2 Multiplication/Division

In a discussion with a group of parents, what came out was that multiplication is not as problematic for the students as division. The slight problem that arises in multiplication is that children get confused with the so-called multiplication rule. According to their textbook the rule says that to find the product of two decimal numbers, we

i) perform the multiplication as if we were multiplying two whole numbers, and

ii) place the decimal point in the product to give as many decimal places in the product as there are in the multiplicand and the multiplier together.

Note the ways in which we make mathematics incomprehensible to the child? To

understand this rule they should first know the terms 'multiplicand and multiplier'.

In spite of the terrible language in which the rule is presented, children usually learn to apply the rule for multiplication or any other rule. But many of them do not remember it for very long, and then they start applying it incorrectly. Let me tell you about an interesting incident here.

Example 5: Once I asked my nephew Nandu, who is studying in Class 6, to multiply 2.14 by 10. He thought for a while and answered, "Is it .214? " Immediately he corrected himself and said, "No, I think, I have to shift the decimal point to the right, so it is 21.4. The fact was that he was not very confident about the rule. He was trying to memorise it without understanding it. I felt that I needed to remedy the situation.

That evening I made a rough plan about the concepts and processes I would explain to him, the pre-requisites and the activities that would help him understand the concepts. Next day we started our discussion.

To start with I checked how he has understood the decimal notation of a number. He seemed to be quite comfortable with the place value representation of a decimal number. Then I asked him

I : Suppose we want to solve 0.1×5. First tell me what 0.1 means to you?

Nephew (N) : Means? In fraction form it is 1/10.

I : O.K. So what is 0.1×5?

He wrote $\frac{1}{10} \times 5 = \frac{5}{10}$.

I : How would you write this in decimal form?

N : 0.5

I : Good. So $0.1 \times 5 = \frac{1}{10} = 0.5$. Now find 0.01×5 and 0.001×5.

N : $0.01 \times 5 = 0.05$.

I : How did you get this?

Nandu wrote $0.01 \times 5 = \frac{1}{100} \times 5 = \frac{5}{100} = 0.05$. And

$$0.001 \times 5 = \frac{1}{1000} \times 5 = \frac{5}{1000} = 0.005$$

I : Yes, very good. Now let me see whether you can find 0.6×0.3 using this.

Nandu wrote $0.6 \times 0.3 = \frac{6}{10} \times \frac{3}{10} = \frac{18}{100} = .18$, and said zero point one eight.

I : So $0.6 \times 0.3 = 0.18$. One more example, what is 0.2×0.14?

Nandu wrote $0.2 \times 0.14 = \frac{2}{10} \times \frac{14}{100} = \frac{28}{1000}$, thought for a moment, and then wrote 0.028.

I : O.K., Now let me collect what we have done so far.

I wrote them down one by one, as below

$0.1 \times 5 = 0.5$

$0.01 \times 5 = 0.05$

$0.001 \times 5 = 0.005$

$0.6 \times 0.3 = 0.18$

$0.2 \times 0.14 = 0.028$

I : Look at all of them. Can you find any relationship between the total number of digits to the right of the decimal point on both sides of the "equal to" sign? Look at the first one. On the left hand side, altogether, how many digits are there to the right of the decimal point?

N : There is only one.

I : What about the right hand side?

N : Only one.

I : What about the next one?

N : There are two digits on the right and two on the left. In the third one there are three on the left, three on the right. And, in the fourth case (he thought for a moment), one digit in 0.6 and one in 0.3. Therefore the total is two and 2 on the right. Next case, there's one in 0.2 and two in 0.14. So altogether three and three on the right.

I : So, do you see a pattern in each of the cases?

N : The total number of digits on the left and right of the "equal to" sign are the same.

I : Anything else? (He was silent).

O.K. Look at 0.6×0.3, is there any relationship between 6, 3 and 18?

N : Yah: 18 is 6 times 3.

I : What about the numbers in the other cases?

N : Yah, I got it. 28, 14 into 2, $1 \times 5 = 5$, like that.

I : Excellent! So you have found the multiplication algorithm for yourself. So, why don't you tell me the rule, step by step.

N : Suppose we want to multiply, 0.7 and 0.6, we first multiply the whole numbers 7 and 6 which give 42. Then count the number of digits to the right of the decimal point in each factor. Here it is two. Therefore, the product is 0.42.

I : Good Now, if you ever get confused while applying this algorithm you can check your answer by rewriting the numbers in fraction form i.e.

$$0.7 \times 0.6 = \frac{7}{10} \times \frac{6}{10} = \frac{42}{100} = 0.42.$$

Do you understand?

N : Yes, Auntie.

: Now, let me come to the problem I asked you yesterday. What is 2.14×10? You were confused about whether it is .214 or 21.4. Why don't you check it?

He wrote $2.14 \times 10 = \frac{214}{100} \times 10 = 21.4$

N : It is 21.4

I : Check it by applying the algorithm.

N : $214 \times 10 = 2140$. There are two digits on the left hand side. Therefore, the number is 21.40.

I : Good. Now you have no confusion, right?

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The situation above illustrates how students can develop the multiplication algorithm for themselves, rather than merely memorising a procedure they do not understand. It also shows that if children have any confusion about applying the algorithm, they can revert to the fraction method and check their answer. This helps to build their confidence in what they are doing.

Now here is a related exercise for you.

E12) How can we use the activity "Dividing the square into 100 parts" to introduce multiplication involving decimal numbers?

Let us now discuss problems related to division with decimal numbers. As we have been stressing again and again the **problems arise mainly due to lack of understanding of the basics involved in the process of division.** It has been pointed out by some teachers that children mainly commit mistakes when the divisor is a decimal number. Let's see how far the fraction method helps in this situation.

Example 6: A teacher of Class 5 was going to help her learners to see how to divide a number by a decimal number. As an example she wrote $24 \div 0.4$ on the board.

Teacher : As we have seen for multiplication, we shall use the fraction method, i.e., convert decimal numbers into fraction form and then use the rule for division for fractions. So, tell me how to convert 0.4 into fraction form

One child : 0.4 is $\frac{4}{10}$.

Teacher : Now can you tell me how to find $24 \div \frac{4}{10}$.

One child : $24 \div \frac{4}{10}$ is $24 \times \frac{10}{4} = 60$.

The teacher did one or two examples of dividing a whole number by a decimal number. Then she explained how to divide a decimal number by another decimal number. For example, $0.36 \div 0.2$. As usual she asked them to convert both decimal numbers to fraction form. The children had no difficulty in writing

$$\frac{36}{100} \div \frac{2}{10}.$$

But, after this step, only a few students could correctly do the division.

Some of them did $\frac{100}{36} \times \frac{10}{2}$.

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Of course, those students who have really understood how to divide one fraction by

another have no difficulty in applying this method. But others find this method difficult. For them it is better to use the equivalent method, that is,

$$\frac{0.36}{0.2} = \frac{0.36 \times 10}{0.2 \times 10} = \frac{3.6}{2}.$$

Since division by a whole number is not a problem, they might find this method easier.

But, even while doing this kind of a division children usually commit mistakes. They tend to be confused about where the decimal point should be placed. For example when I once asked a child studying in Class 6 to solve 54.16 ÷ 8, she did it in the following way:

$$\begin{array}{r} 0.677 \\ 8\overline{)54.16} \\ \underline{48} \\ 61 \\ \underline{56} \\ 56 \\ \underline{56} \\ 0 \end{array}$$

It is clear that she was blindly applying her version of the rule, without understanding the algorithm. Her answer didn't even seem absurd to her, because she had completely forgotten that she was dividing approximately 54 and a portion by 8. She didn't realise that her answer should have been approximately 7.

It is important to help children learn to estimate the answer.

As we pointed out earlier, it is necessary that we should explain to children when to put the decimal point in a division. We stress the fact that the dot separates the whole number part from the fractional. It may help to explain it in the following stages.

Stage 1: First show how to divide, say 4.16, by 8. The idea behind in choosing the numbers 4.16 and 8 is, that the whole number part of 4.16 is 4, which is less than 8. We can replace 4.16 by any other number whose whole number part is less than 8.

Now, to divide 4.16 by 8, we first have to divide 4 by 8. If we do so, we get a number less than 1, which is a proper fraction. So there is no whole number part in the quotient (See Fig.3). We start the quotient with a decimal point to show that there is no whole number part, and the fractional part begins. Then we continue as in Fig.3.

$$\begin{array}{r} .52 \\ 8\overline{)4.16} \\ \underline{40} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Fig. 3

[**Remark:** Here the children should be helped to realise that there is a way for verifying whether the division is right — see whether .52 × 8 is 4.16 or not.]

As a next stage we can give them problems like, 54.16÷8.

Stage 2: Note that we first have to divide 54 by 8 (See Fig.4). Then we get the whole number 6 as the remainder. Next we have to divide 6 by 8. At this stage the whole number part is less than 8. Therefore, we are in the situation given in stage 1, and we should proceed accordingly (See Fig. 4)

$$\begin{array}{r} 6.77 \\ 8\overline{)54.16} \\ \underline{48} \\ 61 \\ \underline{56} \\ 56 \\ \underline{56} \\ 0 \end{array}$$

Fig. 4

Then give lot of practise exercises, each time stress the fact that, whenever there reaches a stage where the whole number part is less than the divisor, we put a dot in the quotient.

After getting enough practice with divisions of this kind, where the numbers which are fully divisible by the divisor, we can go to the next stage.

Stage 3: Dividing, say 54.12 by 8, that is numbers in which zeros have to be attached to the right of the numeral to make it divisible by the divisor.

$$8\overline{)3.12}\quad 0.39$$
$$24$$
$$72$$
$$72$$

Let's now have a look at the synthetic division procedure for, say 3.12 ÷ 8 (see margin). To understand this procedure clearly, may be we should look at what is actually going on

$$3.12 = 3 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{10}$$

In fact, the whole process of synthetic division consists of many sub-processess which **we need to understand** before explaining the process to our learners. Of course, explaining the sub-processes to your learner may confuse her **unless** she is ready to understand the logic involved. For instance, let us see what is actually happening when we divide 3.12 by 8.

$$3.12 = 3 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100}$$
$$= \frac{30}{10} + \frac{1}{10} + 2 \times \frac{1}{100}$$

This explanation should not be given to children till they are old enough to understand the logic, may be after Class 8.

[Since 3 < 8, to divide the quantity by 8, we go to the place next to 3 and treat 3 as $\frac{30}{10}$.

$$= \left(8 \times \frac{3}{10} + \frac{7}{10}\right) + \frac{2}{100} \left(\text{since } \frac{31}{10} = \frac{8 \times 3 + 7}{10}\right)$$
$$= 8 \times \frac{3}{10} + \left(\frac{70}{100} + \frac{2}{100}\right)$$
$$= 8 \times \frac{3}{10} + \frac{72}{100}$$
$$= 8 \times \frac{3}{10} + 8 \times \frac{9}{100}$$
$$= 8 \times 0.39.$$

Unfortunately, most of us don't realise the number of sub-processes involved in the whole process. You might agree with me that only those children who have a strong basis of the place value representation of decimal numbers can understand these sub-processes clearly.

Now, for some exercises related to what you have just read!

E13) Devise a game which could be used to help children improve their understanding of multiplication and division of decimal numbers.

E14) What are the different means that you use for evaluating children's understanding of different operations with decimals?

We end this unit with highlighting a point which we have made again and again; **children learn from anything and everything they experience.** They are more interested in the objects and tools that they use in their daily lives rather than in almost any special learning materials made for them. Their real-life experiences are sufficient to give them plenty of opportunities to use decimal numbers. We should exploit these experiences to pass on the concepts and skills related to decimal numbers.

Let's now summarise what we have covered in this unit.

15.4 SUMMARY

In this unit we have

1) discussed some basic problems that children face while handling decimal numbers.

2) discussed some strategies which can improve their understanding of the decimal representation of numbers.

3) looked at various ways of introducing children to operations on decimal numbers.

4) Stressed the importance of

- concrete based learning
- giving them a variety of activities to do
- helping them see the logic of an algorithm rather than just memorising it blindly.

Throughout this unit we have highlighted the need for the child to properly **understand the now and why** of representing a number in decimal notation. Once this is done, the child usually does not face a problem when operating with decimal numbers.

15.5 COMMENTS ON EXERCISES

1) First check if she can differentiate between 0.6 and 0.06. Next, you could ask her that if she calls 0.06 as zero point 6, then what would 0.6 be? This will set her thinking.

E2) Note that the teacher in Example 1 is directly giving the facts without giving the reason behind them. This method has got the advantage that it is simpler for the teacher and direct. This teacher and some other teacher have observed that children at this stage are not able to understand the reason behind the rules. So the best thing is to let them practise the rule repeatedly, and over a period of time they would understand the reason behind them. But does this method help the child to develop her ability to think mathematically (see Unit 1)?

E3) Think about the quality of long-term learning.

E4) An activity could be "Dividing the square into 100 parts". Draw a large square on the black board. Ask one child to divide the square into ten rows of ten squares (see Fig.5)

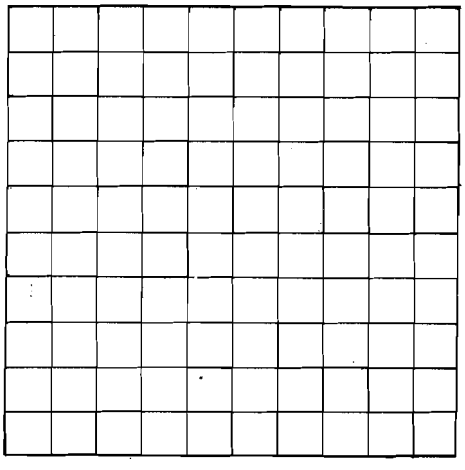
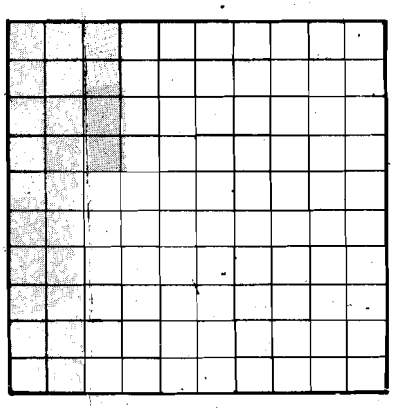


Fig.5

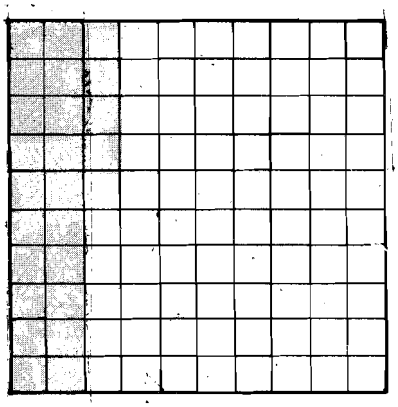
Mark the long thin rectangles that are tenths of a square. This is done so that the children have a visual demonstration that ten hundredth of the square are the same as one tenth of it. We should give ample time for children to identify the shapes that are called tenths and hundredths of the square. Once they have identified the shapes, we can ask them how many hundredths of the square make a tenth of it, how many make two tenths of it and so on. Then we can shade some portion of the square, say the portion corresponding to twenty-four hundredths of the square (see Fig.6(a)) and ask them to colour it with red. Then ask them how many tenth of the square are

there in shaded portion and ask them to colour it with Grey. (see Fig.6 (b)).

Here they identify the shaded portion as 2 tenths and 4 hundredths. By now they know that we can denote 2 tenths and 4 hundredths as .24. Then we can ask children to colour various portions and ask them to write the number corresponding to that in decimal form. Then give them a number and ask them to colour the corresponding tenths and hundredths. Try this activity and see its effectiveness.



(a)
24 hundredths coloured red.



(b)
2 tenths coloured Grey and 4 hundredths, coloured red.

Fig.6

To assess how useful the activity has been in improving the child's understanding of decimal, you could ask her to do exercises like:

i) How to write numbers like **eleven tenths, nine hundredths,** in decimal notation.

ii) Fill in the blank:

Six tenths is the same as ....**hundredths.**

iii) Correctly place the following decimals in order of size, with the smaller first

1.32, 0.89, 3.04, 2.09

E6) False. The child can faultlessly apply the rule without understanding it. Therefore if she is posed a slightly different problem from what she is used to, she may not be able to apply the rule in that situation.

E7) It may be due to the fact that she didn't give them enough practice for writing decimal numbers as multiples of hundred, ten, one, one-tenth, and so on. Does the way given below improve matters?

Step I: First ensure that they can write a decimal number as multiples of ten, one, one-tenth, etc. and vice-versa.

Step II: Then remind them of the procedure for adding whole number by giving an example, 219 + 73. They do this by writing 3 below 9, i.e., the ones' below the 'ones', the 'tens' below the 'tens' and so on. That is, decimal numbers are also added by first writing them 'hundredths' below the hundredths the tenths' below the 'tenths', and so on. For example, suppose we want to add 18.7 and 20.65. We place the 2 below 1, in the tens column, and so on. The learner may wonder what to do about the 5 in 20.65, since there is no corresponding digit in 18.7. Here is where we can tell them that "there are no hundredths in 18.7" is the same as writing 0 in this column, and why 18.7 is the same as 18.70. Now the child can place the 5 below the 0 in the hundredths column.

E8) "Handling accounts while purchasing different materials" is one situation where they may be indirectly applying their knowledge of decimal numbers. But remember that they may easily do these additions because of their familiarity with handling money. They may not use their understanding of decimal notation at all.

Did any child think of the following situation?

She may want a skirt and a blouse for herself. If the skirt requires 3m and 50cm of cloth and the blouse needs 1m and 70cm, then she needs to know how much cloth of the same material she should buy. Do you think that she is applying her knowledge of decimal notation? The point is that the situations should be such that they really require their knowledge of decimal notation to perform the operation.

E9) In E4 we have already explained how to represent a decimal number in a square divided into 100 parts. To start with consider only those numbers whose sum is less than 1. We can use the same square to introduce sums of the form, say 0.31 + 0.25. First we can ask them to mark 0.31 on the square, ask them to colour the tenths by shading and hundredths by red colour (see Fig.3), then ask them to represent 0.25 on the square, by showing the number of tenths and hundredths. Then we can ask them to count the total number of tenths and hundredths. They can see that there are 6 hundredths and 5 tenths which gives the sum. As a next step, you can ask them to add 0.18 + 0.23. Here they have to use the conversion 10 hundredths = 1 ten. Next we can give problems like 0.06 + 0.25, 0.7 + 0.04.

After they get enough practice for doing the sums using the square, we can introduce "vertical addition" side by side using the square. Remember that we are considering numbers whose whole number part is 0. Gradually we can introduce the sums of the form, 18.7 + 20.65.

Don't you think that those students who have been trained to represent decimal numbers in a square divided into 100 parts" will find this method much easier? The advantage of this method is that, intuitively this gives an idea that we are adding tenths, hundredths, tens and ones etc. Initially children may get slightly confused with the relationship between tenths and hundreds. Once that is clear, then this method may work better. Why don't you try it and see its effectiveness?

E10) Each of the three errors is a way of the child trying to cope with the lack of digits in the hundredth's place in 3.4.

a) The child committed this error, probably due to this fact : Even if she knows that it should be 3.4 in the first line and 1.53 in the second line, she gets confused by the lack of a digit above the 3 in the number 1.53, so she gets confused and interchanges the numbers.

As a remedy, what I would have done is to first ask her to add 3.4 and 11.9 and see whether she gets 1.53. This would help to convince her that she has gone wrong. Then I would help her to develop an understanding of the place value representation of decimals. This in turn, would help her to cope with the missing digit in the hundredth's place of the number 3.4.

(b) and (c), you can handle similarly. You can write down several teaching strategies for remedying this.

E11) Here we shall briefly explain how we can use square charts to perform subtraction, say 3.4 – 1.53. In a big chart you draw four squares divided into 100 parts and colour the portion corresponding to 3.4 on that by red (See Fig.7 (a)).

Now delete 1.53 from 3.4 by removing the red coloured portion corresponding to 1.53 from the portion showing 3.4 (see Fig.7 (b))

You can give enough practice to students for handling subtraction with these squares. Then you can show subtraction by placing the numbers vertically, say 3.4 – 1.53, and along with that you ask them to check their answer with the square.

The advantage of this is that even if they have some confusion in the direct method, they can revert to the activity and do the same problem with the activity. This will help them to gain confidence in applying the algorithm.

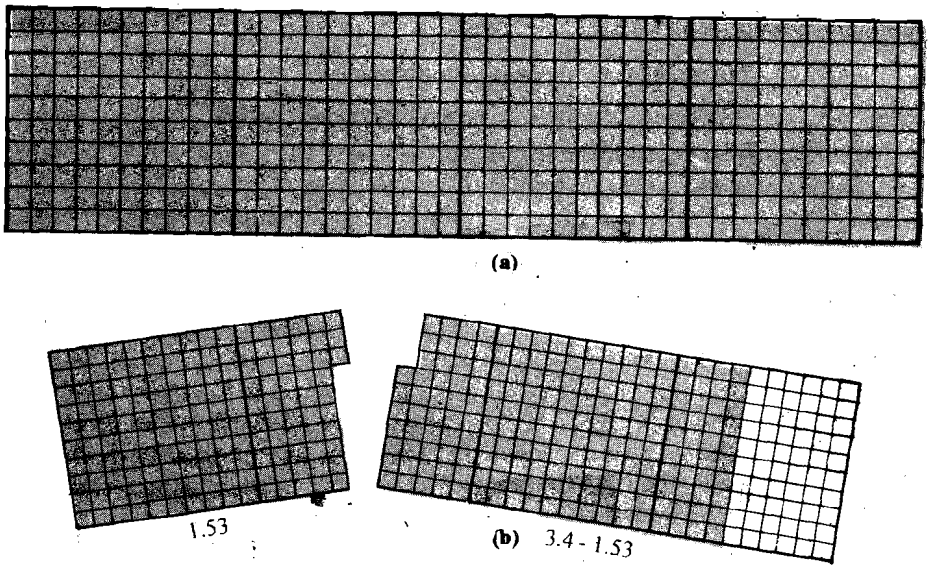


Fig.7: Red coloured portion (a) shows 3.4 (b) shows 3.4–1.53

You could try this experiment on children around you and see whether this really helps.

E12) To start with you can explain to them the multiplication $0.1 \times 0.1 = 0.01$. First of all you should ask them to convert the decimals into fractions, that is

$$.1 \times .1 = \frac{1}{10} \times \frac{1}{10}$$

Now, $\frac{1}{10} \times \frac{1}{10}$ means $\frac{1}{10}$ of $\frac{1}{10}$ of a whole. Now to find $\frac{1}{10}$ of whole, consider a square divided into 100 parts. First mark the portion corresponding to $\frac{1}{10}$. You will mark a strip as shown in Fig.8 – colour it red

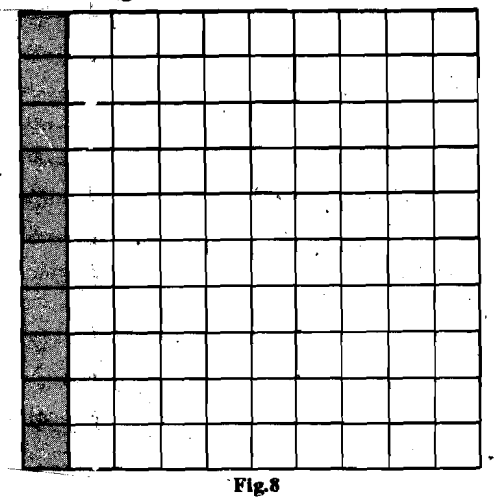


Fig.8

To see that it is $\frac{1}{100}$, double shade $\frac{1}{10}$th of red coloured portion by gray which can then be related to $\frac{1}{100}$.

Therefore, $.1 \times .1 = \frac{1}{10} \times \frac{1}{10} = .01$

Next you can show simple multiplication whose product is less than 1, say $.2 \times .3$. As in the above case turn into fractions. This will give that $.2 \times .3 = \frac{2}{10} \times \frac{3}{10}$. First shade $\frac{3}{10}$ in a square divided into 100 parts (see Fig.9), colour it red

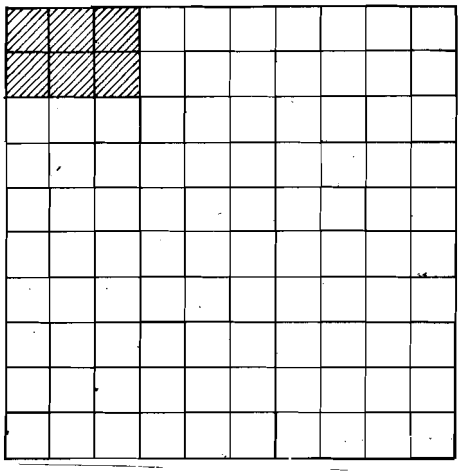


Fig.9

Now find $\frac{2}{10}$ of the red coloured portion. To find that we first have to divide that portion into 10 parts and consider 2 parts from that. In Fig.9 we have divided the coloured portion into 10 parts and shaded 2 parts as Grey. Hence the Grey coloured portion shows $\frac{2}{10} \times \frac{3}{10}$. Now note that there are 6 small squares in that portion which corresponds to the decimal number .06. Hence we have shown that $.2 \times .3 = .06$.

E13) Suppose there are 4 children, you can make a game like the following:

You can prepare a set of 25 cards with multiplication and division questions on one side and answers on the reverse. The answer to one question is written on the back of the card with the next question. The answer to the question on the last card in the set is written on the reverse of the first card. There should not be any repetition in the answers (see Fig.10 below)

	0.1 × 0.1	0.3 × 0.6	0.4 × 10	0.06 × 2	0.2 × 10	0.12 ÷ 0.4	0.5 ÷ 0.1	.6 ÷ 0.2
Reverse of the above cards	3.3	0.01	0.18	4	0.12	0.02	0.48	5
	5 × 0.2	0.8 × 0.01	4 × 0.8	8 ÷ 0.4	0.1 × 5	2 ÷ 0.2	0.3 × 0.3	0.07 × 10
Reverse of the above cards	3	1	0.008	3.2	20	0.5	10	0.09
	9 ÷ 0.1	0.01 × 6	0.1 ÷ 1	0.9 × 0.04	0.15 ÷ 0.2	0.14 × 10	2.5 ÷ .01	0.36 ÷ 0.6	11 × 0.6
Reverse of the above cards	0.7	90	0.06	0.1	.036	3	1.4	0.25	0.06

Fig.10

Now you shuffle the cards and distribute those equally among four children, say. You place the remaining cards in the centre of the table, question uppermost. If for example, the question is 0.3×0.6, then you ask who has got the answer for this.

Whoever has the card with 0.18 on the reverse then places it next to the card, on which it is written 0.3×0.6. When all the students agree with the answer, you turn this card over to reveal the next question which is 0.4×10, and asks who has got the answer for 0.4×10. The game proceeds like this till the students get rid of all their cards.

Try this game on children around you and see its effectiveness.

E14) One way to evaluate student's understanding of number operations is to ask them word problems of the following kind.

a) For the following questions **ring the calculation** you would need to do to find the answer, then do the calculation.

For example, suppose that the question is like this:

Raju earns Rs. 1.45 an hour delivering newspapers. How much can he earn in 5 hours.

i) $1.45 + 5$ ii) $5 - 1.45$

iii) 1.45×5 iv) $\frac{1.45}{5}$

Now, $1.45 \times 5 = 7.25$

Therefore, the answer is 7.25

Similarly, you can ask them to answer the following questions.

b) A car can go 15.80 kilometres on one litre of petrol. How many kilometres can it go with 70.50 litres of petrol?

i) $15.80 + 70.50$ ii) $15.80 - 70.50$

iii) 15.80×70.50 iv) $70.50 \div 15.50$

b) The cost of 3.4 metres of a cloth is Rs. 40. What is the price of 1 metre of the cloth?

i) $40 - 3.4$ ii) $3.4 - 40$

iii) 40×3.4 iv) $40 \div 3.4$

c) Ring the bigger number in each of the following cases.

i) 5×0.4, $5 \div 0.4$

ii) 0.4×0.8, $0.8 \div 0.4$

iii) 0.6×3, $3 \div 0.6$

iv) 0.7×0.5, $0.7 \div 0.5$